OpenSees-SNOPT Framework for Finite-Element-Based Optimization of Structural and Geotechnical Systems

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Abstract: The finite-element (FE) method is widely recognized as a powerful tool in modeling structural and geotechnical systems and simulating their response to static and dynamic loads. In addition, numerical optimization is commonly used in many engineering applications, such as structural reliability analysis, FE model updating, structural identification, and structural optimization. This paper focuses on the extension of Open System for Earthquake Engineering Simulation (OpenSees, an existing software framework for nonlinear FE analysis) using Sparse Nonlinear Optimization (SNOPT, a state-of-the-art numerical optimization software). The extended OpenSees-SNOPT framework is general and flexible and can be used to solve a wide range of FE-based optimization problems in structural and geotechnical engineering. It has several distinguishing features: (1) advanced capabilities in solving optimization problems involving complex structural/geotechnical engineering systems; (2) versatility in modeling a very wide range of structural and/or geotechnical systems; (3) computational efficiency; (4) flexibility to easily accommodate and benefit from new developments in FE structural modeling and analysis, computational optimization, and probabilistic modeling and analysis; and (5) capabilities of exploring new optimization-based problems and solution methods. The use of this coupled framework is illustrated through three representative application examples, i.e., a FE reliability analysis of a reinforced concrete frame, a FE structural optimization problem of an electrical transmission steel tower, and a FE model updating the problem of a geotechnical system. **DOI: 10.1061/(ASCE)ST.1943-541X.0000511.** © *2012 American Society of Civil Engineers.*

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Introduction

The finite-element (FE) method is a powerful tool used to model and simulate the response to static and dynamic loads of structural, geotechnical, and soil-structure interaction (SSI) systems typically encountered in civil engineering. At the same time, numerical optimization is commonly used to solve a number of classes of problems in civil engineering, such as nonlinear FE model updating, structural optimization, and reliability analysis. The combination of FE and numerical optimization methods, referred to herein as FE-based

optimization, provides a practical tool for improving the design, assessment, and retrofit of new and existing structures.

The development of FE-based optimization software has been the object of intense research, particularly for mechanical engineering applications. The optimization software TOSCA (Parker et al. 2003) and the FE analysis software ABAQUS (HKS 1997) were integrated and employed in industrial applications of topology and shape optimization for mechanical design, e.g., lightweight design of automotive and railway components (Sauter and Meske 2001). Other FE analysis software—for example, Nastran (MSC 2007), ANSYS (Kohnke 1998), and LS DYNA (LSTC 2007)—also includes numerical optimization tools enabling the users to optimize structural designs of mechanical components, with particular emphasis on weight reduction through optimization (Johnson et al. 2004; Goel et al. 2009).

Optimization problems in civil engineering are very complex in nature and stem from a broad range of applications. These problems can involve FE response of structural, geotechnical, or SSI systems to various static and/or dynamic loads and can require optimization of different system properties and/or system response behavior (e.g., modal frequencies, damping properties, mode shapes, moment-curvature relationships, force-deformation relationships, and various features of displacement/velocity/acceleration response histories). The quantities defining the objective function(s) to be minimized (e.g., initial cost, life cycle cost, reliability index, total weight, and any linear or nonlinear combination thereof) and the constraints to be satisfied (e.g., maximum displacement/acceleration/ stress responses, maximum plastic deformation, geometric or manufacturing constraints, and reliability index) can also significantly differ from application to application. There is a clear need for a FE-based optimization framework that is sufficiently general

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and flexible to accommodate the wide range of optimization problems encountered in civil engineering. In addition, this FE-based optimization framework must be able to incorporate, without major modifications, current and future advances in linear and nonlinear FE analysis and computational optimization. In this context, initial efforts have been made to combine numerical optimization software and advanced nonlinear FE analysis programs for the purpose of performing FE-based optimization (Liang et al. 2007).

This paper presents the extension of the general purpose software framework Open System for Earthquake Engineering Simulation (OpenSees) (Mazzoni et al. 2007) to enable FE-based optimization in civil engineering. This extension is obtained by coupling OpenSees with Sparse Nonlinear Optimization (SNOPT) (Gill et al. 2005), which is a well-known state-of-the-art numerical optimization software, through object-oriented programming concepts. This extended OpenSees-SNOPT framework has several distinguishing features: (1) it is very powerful, benefitting from the extensive computational simulation capabilities of OpenSees and the remarkable efficiency of SNOPT in solving high-dimensional numerical optimization problems; (2) it is very versatile because it allows a wide range of problems to be solved, including nonlinear FE model updating, structural optimization, and reliability analysis of structural, geotechnical, and SSI systems; (3) it achieves computational efficiency by building on the individual efficiency of each of the two programs and ensuring an efficient communication between them; (4) it has the flexibility to easily accommodate and benefit from new developments in FE structural modeling and analysis, computational optimization, and probabilistic modeling and analysis; and (5) it enables the easy exploration of new optimization-based problems and solution methods.

The OpenSees-SNOPT framework allows the users to implement the specific objective and constraint functions required for the application of interest by using Tool Command Language (TCL) (Ousterhout 1994). TCL is the scripting language upon which the OpenSees user interface is based. By exploiting the flexibility of OpenSees together with the powerful features of TCL, the user can employ the OpenSees-SNOPT framework for any application requiring FE-based optimization by using standard functions defined in a TCL library or by customizing the TCL commands to define objective and constraint functions not included in the library. In this coupled framework, the objective and constraint functions can be expressed in terms of any model parameter used to build a FE model in OpenSees and any system property and/or response parameter obtained from a FE analysis performed in OpenSees.

The objectives of this paper are as follows: (1) describe the object-oriented implementation of the SNOPT extension of the existing OpenSees framework into the so-called OpenSees-SNOPT framework and (2) illustrate the capabilities of this extended framework. The first objective is the motivation for the thorough description of the implementation, which is intended to provide the information necessary for other researchers to extend the framework. The second objective provides the justification for the description of the state-of-the-art applications of the extended OpenSees-SNOPT framework.

Finite-Element Analysis Component: OpenSees

OpenSees is an open-source object-oriented FE analysis software framework, implemented using the C++ programming language. OpenSees can be used to model structural, geotechnical, and SSI systems and to simulate their response to static and dynamic loads, with particular emphasis on earthquake loads (McKenna et al. 2010). OpenSees has been developed under the auspice of the Pacific Earthquake Engineering Research Center, is currently the

main simulation platform for NEESGrid (http://www.nees.org), and is widely used in the academic world. OpenSees supports a wide range of simulation models, solution procedures, and distributed computing capabilities. Unique among software for earthquake engineering, OpenSees allows the integration of models of structures and soils to investigate challenging problems in soil-foundation-structure interactions (Jeremić and Yang 2002; Boulanger et al. 2003; Jeremić 2004; Wang and Sitar 2006; Elgamal et al. 2008; Gu 2008; Lam et al. 2009). OpenSees provides advanced numerical models for reinforced concrete and steel structures, shallow and deep foundations, and liquefiable soils. OpenSees also takes advantage of the latest developments in databases, scientific visualization, and high-end computing. Furthermore, OpenSees includes modules for FE response sensitivity analysis and FE reliability analysis (Haukaas and Der Kiureghian 2005; 2006). In particular, the FE response sensitivity analysis module, based on the direct differentiation method (DDM) (Zhang and Der Kiureghian 1993; Kleiber et al. 1997; Conte et al. 2003; Haukaas and Der Kiureghian 2006), provides an accurate and efficient tool for response gradient computation (with respect to material, geometric, and loading parameters), which is extremely useful in computational optimization problems. The superior performance of the DDM over the finite-difference method in stand-alone sensitivity analysis and in applications involving numerical optimization has been shown in numerous recent studies, the majority of which were performed in OpenSees (Gu and Conte 2003; Conte et al. 2004; Scott et al. 2004; Zona et al. 2005; 2006; Haukaas and Scott 2006; Barbato et al. 2007; Scott and Filippou 2007; Gu 2008; Gu et al. 2009a,b). OpenSees classes can be classified into three categories: (1) domain classes, which encapsulate the FE model and correspond to domain component objects (e.g., element, node, load pattern, and single and/or multipoint constraint objects); (2) analysis classes, which include classes responsible for performing a fundamental operation in determining the state of the FE model (e.g., solution algorithm, equation solver, integrator); and (3) model builder classes, which populate the domain classes based on user input (McKenna et al. 2010). Introduction of new modules into OpenSees requires implementation of classes in all three aforementioned categories.

Numerical Optimization Component: SNOPT

SNOPT is a software package, implemented using the Fortran programming language, used for the efficient solution of large-scale linear and nonlinear, unconstrained and constrained numerical optimization problems. SNOPT uses a sequential quadratic programming (SQP) algorithm to find local optima of problems defined with smooth objective and constraint functions. SQP algorithms generate an iterative sequence in which each iteration is based on an approximate solution of a quadratic programming (QP) subproblem defined with a quadratic model of the Lagrangian function and linearized versions of the nonlinear constraints. The SQP method used in SNOPT has several features that are useful for civil engineering applications:

- SNOPT is specifically designed for large-scale applications. The algorithm exploits sparsity in the Jacobian matrix of the constraint functions and uses a limited memory quasi-Newton method to avoid the calculation of the Hessian matrix of second derivatives of the Lagrangian function. The QP subproblems are solved using an advanced inertia-controlling reduced-Hessian active-set method (Gill et al. 2005) that is designed for large-scale problems with several thousands of variables and constraints. The method is particularly efficient when solving problems with relatively few degrees of freedom (defined as

the difference between the total number of optimization variables and the number of constraints satisfied with equality at a solution).

- Compared with software that does not use a SQP approach, SNOPT needs relatively few evaluations of objective/constraint functions and their gradients. This is crucial for the efficient solution of FE-based optimization problems because the evaluation of the objective and constraint functions and their associated gradients requires the computation of the FE response and response sensitivities. This is the most computationally intensive part of the calculation.
- SNOPT tolerates discontinuities in the gradient of objective and constraint functions that are not in the neighborhood of an optimal solution. Discontinuities in the gradient of objective and constraint functions correspond to discontinuities in the sensitivities of the FE response with respect to various modeling parameters (e.g., discrete loading, geometric properties, and material constitutive parameters). These discontinuities can be produced by the nonsmoothness of the material constitutive models and/or by relatively large time integration steps (Gu and Conte 2003; Barbato and Conte 2006).
- SNOPT can avoid regions where the objective and constraint functions are either not defined or cannot be evaluated. Nonconvergence of the numerical integration scheme in computing the FE response can result in large regions of the parameter space where the objective and/or constraint functions cannot be computed. SNOPT is able to recognize these regions and steer away from them during the optimization process. This feature makes SNOPT a robust tool for FE-based optimization problems.
- SNOPT is designed to be flexible by allowing the user to customize the values of certain parameters that can be chosen to improve the efficiency of the optimization for specific problems.

OpenSees-SNOPT Framework for FE-Based Optimization

The newly developed FE-based optimization framework is obtained by linking OpenSees and SNOPT, as shown in Fig. 1. This coupled framework has been implemented based on an object-oriented approach by extending the capabilities of the FE software framework OpenSees. The implementation of this coupled framework is consistent with the framework for parameter updating in OpenSees (Scott and Haukaas 2008). An abstract analysis class named Optimization is introduced. In combination with OpenSees, numerical optimization can be used for various purposes and concrete subclasses need to be implemented to define the particular application (e.g., structural optimization and structural reliability analysis) and the specific numerical optimization tool to be used. Currently, only one subclass of Optimization is available, namely the subclass SNOPTClass, which is used to encapsulate the SNOPT software. However, other numerical optimization software can be easily added by inheritance from Optimization. It is noteworthy that several optimization algorithms for the design point (DP) search in structural reliability analysis are already an integral part of the reliability analysis module (Haukaas and Der Kiureghian 2005, 2006).

SNOPTClass is a concrete class that implements in OpenSees the interface for numerical optimization analysis based on SNOPT. Among the class members that define this interface, four member variables (i.e., pointers to the design variable vector **DV**, objective function F, constraint function vector **G**, and function OptimManager), and a member function [i.e., solve()] are particularly important. The function solve() calls the SNOPT library to perform the numerical optimization analysis. The member pointer OptimManager points to a global function that needs to be implemented for each subclass of SNOPTClass for specific optimization purposes. This global function is called by SNOPT at each iteration of the optimization process to compute and return, for each set of **DV** values, the values of F and **G** and their gradient with respect to **DV**. It is noteworthy that **DV**, F, and **G** also need to be defined for each of the subclasses of SNOPTClass and, thus, for different optimization applications (e.g., structural optimization, DP search, and reliability-based optimization).

Currently, two subclasses of SNOPTClass are implemented in OpenSees: (1) SNOPTReliability, for solving the optimization problem corresponding to the DP search in reliability analysis and (2) SNOPTOptimization, for solving general-purpose FE-based
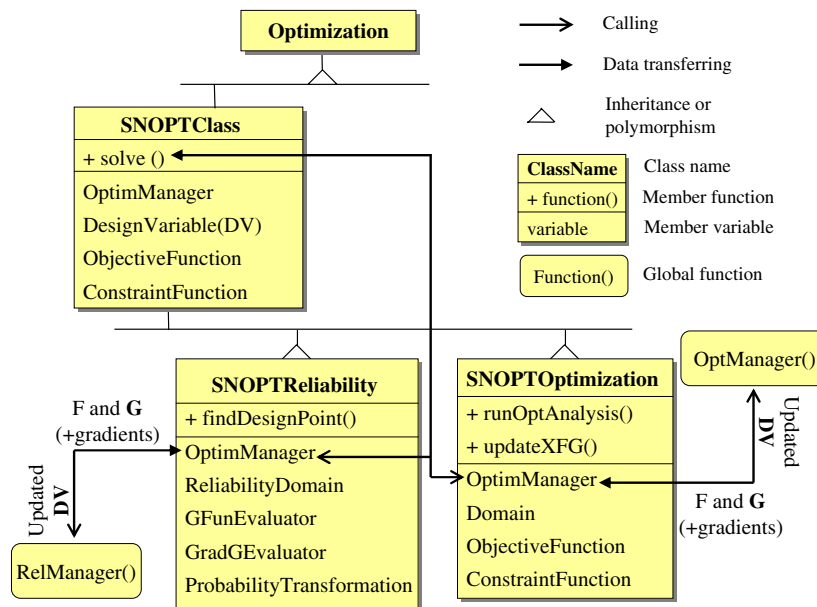

Fig. 1. Optimization framework in OpenSees

optimization problems such as those encountered in structural optimization and FE model updating. Work is underway to implement a third subclass to extend the capabilities of the OpenSees-SNOPT framework to reliability-based optimization analysis. Figs. 2 and 3 show the relations among newly implemented and existing classes in OpenSees for structural reliability applications and general-purpose optimization, respectively. Figs. 4(a) and 4(b) show the flowcharts corresponding to structural reliability applications and general-purpose optimization, respectively. The proposed framework architecture is unique among existing FE programs that include optimization capabilities (which usually include only stand-alone optimization or only optimization for structural reliability applications). Developing this framework architecture is significantly facilitated by the object-oriented software architecture of OpenSees.

SNOPT-Based Optimization Used in Structural Reliability Analysis as a DP Search Tool

In structural reliability analysis, the search of the DP(s) is a crucial step used to evaluate the failure probability by several classical methods, such as the first-order reliability method (FORM), second-order reliability method (SORM), and importance sampling (IS) with the sampling distribution centered at the DP(s) (Ditlevsen and Madsen 1996). These methods use a one-to-one probability transformation between the physical space of random variables (**RV**) and the standard normal space of transformed random variables **Y**. The DP, $\mathbf{Y}^*$, is defined as the most likely failure point in the standard normal space and is usually found as the solution of the following constrained optimization problem (Ditlevsen and Madsen 1996):
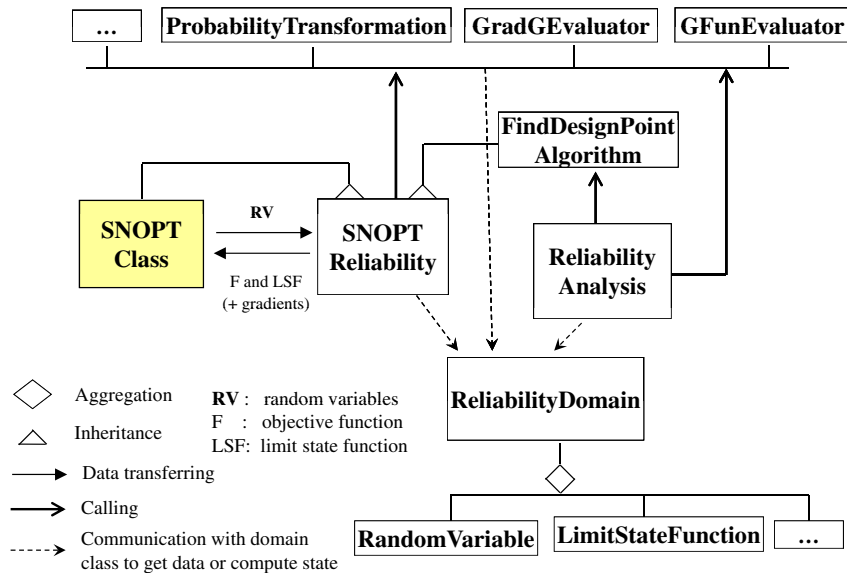

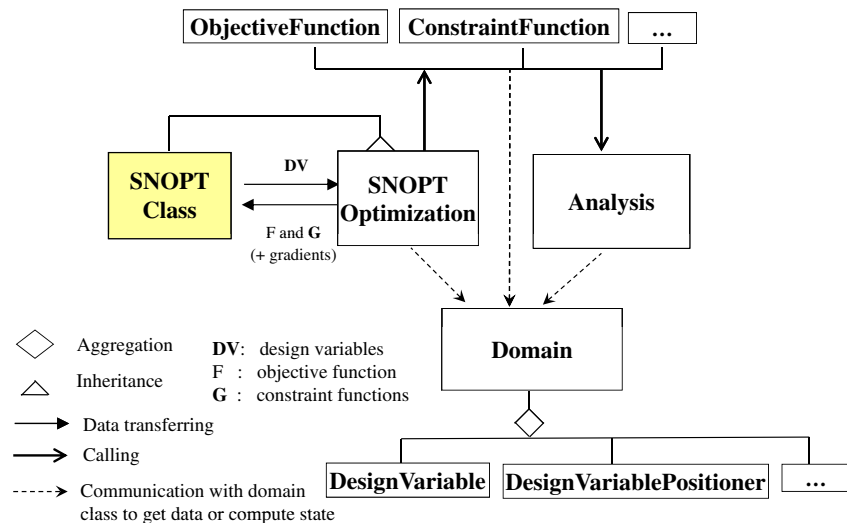**Fig. 2.** SNOPT-based optimization used for structural reliability analysis in OpenSees

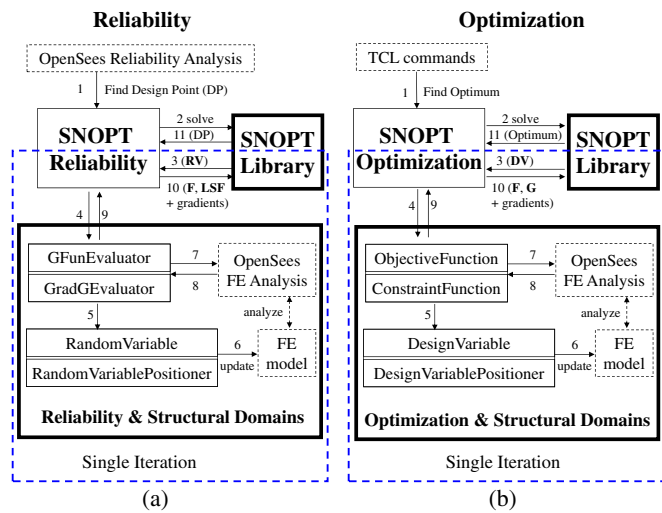**Fig. 3.** SNOPT-based optimization used for model updating or structural optimization/identification

Fig. 4. Flowcharts for the OpenSees-SNOPT FE-based optimization framework

$$\mathbf{Y}^* = \arg\min\left\{\frac{1}{2}\mathbf{Y}^T\mathbf{Y} : G(\mathbf{Y}) = 0\right\} \qquad (1)$$

in which $G(\mathbf{Y}) = $ limit state function (LSF) in the standard normal space, playing the role of the constraint function; and $F(\mathbf{Y}) = (1/2)\mathbf{Y}^T\mathbf{Y} = $ objective function. Gradient-based optimization algorithms, coupled with algorithms for accurate and efficient computation of the gradient of the constraint function $G(\mathbf{Y})$ with respect to $\mathbf{Y}$ are among the most effective techniques used in solving the constrained optimization problem in Eq. (1) (Gill et al. 1981; Liu and Der Kiureghian 1991).

The subclass SNOPTReliability is the specific implementation of the class SNOPTClass that performs the DP search. The member function findDesignPoint() provides the interface between the DP search (in the reliability module of OpenSees) and the optimization component of the framework (i.e., SNOPTReliability), and it calls the function solve() in the base class. The function solve() solves the specific optimization problem by employing the SNOPT library, which requires the values of the objective function $F(\mathbf{Y})$ and constraint function $G(\mathbf{Y})$ and their gradients with respect to $\mathbf{Y}$ from OpenSees at each iteration of the optimization process. These values are computed by using the global function RelManager(), which is the specific implementation of the function pointed to by the pointer OptimManager in the SNOPTReliability class (Fig. 1). In the reliability analysis application of the optimization framework presented here, the vector of design variables $\mathbf{DV}$ consists of the vector of all random variables $\mathbf{RV}$ defined in the physical space. As shown in Fig. 2, the SNOPTReliability subclass inherits both SNOPTClass, which provides the objective and constraint functions needed to perform the numerical optimization to SNOPT, and the existing class FindDesignPointAlgorithm, which provides the interface for the DP search as already defined in OpenSees (Haukaas and Der Kiureghian 2006; Der Kiureghian et al. 2006). Thus, SNOPTReliability can be called by any subclass of the ReliabilityAnalysis class to perform the DP search for any reliability method requiring the DP (e.g., FORM, SORM, and IS). By calling the function RelManager() (see Fig. 1), SNOPTReliability can communicate with the existing domain classes (e.g., RandomVariable and LimitStateFunction) and analysis classes (e.g., ProbabilityTransformation, GFunEvaluator, and GradGEvaluator), which are part of the framework for reliability analysis currently available in OpenSees (Haukaas and Der

Kiureghian 2005, 2006; Der Kiureghian et al. 2006). The detailed flowchart for the SNOPT-based optimization used as a DP search tool is shown in Fig. 4(a). When the OpenSees reliability analysis module is called to find a DP, the SNOPTReliability subclass can be used to perform this search [Fig. 4(a), Step 1]. SNOPTReliability invokes the SNOPT library to find the DP by an iterative process [Fig. 4(a), Steps 2–11]. At each iteration, the SNOPT library updates vector $\mathbf{RV}$ and passes it to SNOPTReliability, which organizes the work of several other classes (i.e., GFunEvaluator and GradGEvaluator in Step 4, and RandomVariable and RandomVariablePositioner in Step 5) to update the FE model (Step 6), perform the FE analysis (Step 7), and retrieve the needed FE response quantities as well as their sensitivities when available (Step 8). Finally, SNOPTReliability evaluates the objective function F, the LSF G and their gradients (Step 9) and passes their values to the SNOPT library (Step 10), which checks for convergence of the optimization process. If convergence is not achieved, the SNOPT library begins a new iteration; otherwise, the DP has been found (Step 11). It is important to mention that FE response sensitivity algorithms based on the DDM are available in OpenSees for a number of material constitutive models and finite elements, and can be used by the presented framework for accurate and efficient computation of the gradient of the LSF G. The DDM computation of the FE response sensitivities greatly improves the robustness and efficiency of the DP search, compared with the FE response sensitivity computation by finite difference.

SNOPT-Based Optimization Used for General-Purpose FE-Based Optimization

FE-based optimization has a number of applications in civil engineering, such as structural optimization and FE model updating. These problems can usually be expressed in the following form:

$$\mathbf{DV}^* = \arg\min\{F(\mathbf{DV}) : \mathbf{LB} \leq \mathbf{G}(\mathbf{DV}) \leq \mathbf{UB}\} \qquad (2)$$

where $F = $ objective function, $\mathbf{G} = $ vector of constraint function, and $\mathbf{LB}$ and $\mathbf{UB} = $ lower and upper bounds on the constraint function.

The subclass SNOPTOptimization is implemented for general-purpose FE-based optimization problems. The member variables of SNOPTOptimization include pointers to the objects instantiating the existing class Domain and two newly developed domain classes (i.e., ObjectiveFunction and ConstraintFunction) used to evaluate the objective and constraint functions for a given set of $\mathbf{DV}$s (Fig. 1). The member function runOptAnalysis() provides the interface between the optimization analysis component and the FE analysis component, and it calls the function solve() in its base class. Similar to the case of the aforementioned DP search, the function solve() solves the specific optimization problem by invoking the SNOPT library, which requires from OpenSees the values of the objective function, $F(\mathbf{DV})$, and constraint functions, $\mathbf{G}(\mathbf{DV})$, and their gradients with respect to $\mathbf{DV}$ at each iteration of the optimization process. These values are computed by using the global function OptManager(), which is pointed to by the pointer OptimManager in SNOPTOptimization (Fig. 1). To perform FE-based optimization analysis, four new classes are created and added into OpenSees (Fig. 3): (1) the domain class DesignVariable, which sets, stores, and updates the values of $\mathbf{DV}$; (2) the domain class DesignVariablePositioner, which maps the $\mathbf{DV}$ vector into the corresponding geometric, material, and/or loading properties of the FE model; (3) the analysis class ObjectiveFunction; and (4) the analysis class ConstraintFunction. By calling the function OptManager() (see Fig. 1), SNOPTOptimization can communicate with the new domain classes (DesignVariable and DesignVariablePositioner) and analysis classes (ObjectiveFunction and ConstraintFunction).

OptManager() also calls the function updateXFG() (member function of SNOPTOptimization), which updates the values of **DV**, calls OpenSees to perform the FE analysis, and then updates the values of *F* and **G** at each iteration of the optimization process. The detailed flowchart for the SNOPT-based optimization used for general-purpose FE-based optimization is shown in Fig. 4(b).

As a distinguishing feature of the OpenSees-SNOPT framework presented here, in addition to a basic library of *F* and **G** functions included in the code, users are allowed to provide their own TCL input file to define the objective and constraint functions, which renders this framework very flexible and versatile. User-defined input files are run by the TCL interpreter, which computes the values of the *F* and **G** functions and of their gradients by using the FE response and response sensitivity analysis results obtained from OpenSees. This implementation allows the use of any FE response result computed by OpenSees in the definition of the objective and constraint functions. Again, in this case, the DDM-based FE response sensitivity analysis capabilities of OpenSees are available and can be used to improve the efficiency of the optimization process. The option to compute the FE response sensitivities by finite-difference analysis is always available.

Advantages of the New Extended OpenSees-SNOPT Framework

The new extended OpenSees-SNOPT framework presented in this paper has several desirable features, which are briefly highlighted in the following.

Feature 1
The framework provides advanced capabilities in solving complex civil engineering optimization problems. These capabilities derive from the combination of the state-of-the-art numerical optimization techniques used in SNOPT (particularly suited for large-scale nonlinear constrained optimization problems), and the cutting-edge resources for linear and nonlinear FE analysis offered by OpenSees (including static and dynamic FE response and response sensitivity analysis of structural, geotechnical, and soil-foundation-structure interaction systems based on sequential or parallel computing). Because of the object-oriented nature of OpenSees, most of these resources are incorporated naturally into the extended OpenSees-SNOPT framework. Other advanced capabilities of OpenSees, such as parallel/distributed computing, require additional (usually limited) programming work to be supported.

Feature 2
The framework is computationally very efficient. In fact, SNOPT needs relatively few evaluations of the objective and constraint functions compared with other numerical optimization software (Gill et al. 2005). This feature is particularly important when these function evaluations are computationally expensive, as in the case of FE-based optimization. In addition, OpenSees provides DDM-based FE response sensitivity analysis capabilities, which can also be important in ensuring fast convergence of the optimization process and are computationally less expensive and more accurate than sensitivities computed using the finite-difference method. DDM-based FE response sensitivities can improve the convergence rate of the optimization algorithm, compared with finite-difference sensitivities, because they are derived through consistent linearization of the FE response algorithms. Finally, the communication between SNOPT and OpenSees is also made very efficient by taking advantage of the object-oriented framework of OpenSees. This is obtained by wrapping SNOPT into an optimization class, which is integrated seamlessly into OpenSees and can interact effectively with any other OpenSees class.

Feature 3
The framework is versatile and can solve a wide range of optimization problems involving structural and/or geotechnical systems. SNOPT is a general-purpose numerical constraint optimization framework that allows the user to specify many optimization control parameters (e.g., feasibility and optimality tolerances, maximum iteration numbers, automatic gradient check switch, size of parameter perturbation) to improve the performance of the algorithm for a given optimization problem (e.g., linear versus nonlinear optimization, constrained versus unconstrained optimization, low- versus high-dimensional parameter space). This freedom for user access to these optimization control parameters is retained also in the proposed FE-based optimization framework. OpenSees supports linear/nonlinear static/dynamic FE analysis of a wide variety of structural, geotechnical, and SSI systems. In addition, the proposed framework allows modeling as a random/design variable any parameter used to define the FE model in OpenSees (e.g., material properties, damping properties, geometric properties, boundary conditions, global seismological parameters, and parameters describing the energy and frequency content of the seismic loading). Furthermore, any FE response quantity computed in OpenSees (e.g., modal frequencies, vibration mode shapes, displacement, velocity and/or acceleration time histories, local force-deformation and/or moment-curvature responses, maximum forces and stresses, maximum displacements, rotations and strains, or any linear or nonlinear combination of the above response quantities) can be used to define the objective and constraint functions for a given optimization problem. Finally, the gradients of the objective and constraint functions with respect to the random/design variables may be computed by using the finite-difference method or the DDM.

Feature 4
The presented framework has the flexibility to readily absorb new developments in (1) computational structural and soil mechanics, (2) nonlinear FE analysis solution strategies, (3) computational optimization, and (4) probabilistic modeling and analysis.

Feature 5
The framework enables the user to easily explore new optimization-based problems and solution methods. This is achieved by allowing users to provide their own TCL input file to define the objective and constraint functions and their own analytical gradients with respect to the demand- and capacity-type variables used to formulate these functions. In this way, objective and constraint functions (e.g., linear and nonlinear combinations of input parameters and response quantities obtained from OpenSees) of unlimited complexity can be easily implemented, providing the users with the capability of freely defining their own specific optimization problem. Furthermore, by using the capabilities of TCL, the presented framework can readily call other existing software or user-defined executable programs for computing the values of the objective and constraint functions and of their gradients. This specific feature has been successfully tested by the writers when employing MATLAB and its signal processing toolbox to transform the time-domain response obtained from OpenSees in the time domain to the frequency domain.

In practice, SNOPT will usually converge to a global solution of the optimization problem. Nevertheless, SNOPT is a local optimization method, which implies that the proposed framework cannot be guaranteed to provide a global solution. The framework may be extended to incorporate a global optimization algorithm, such as the method of coupled local minimizers (Suykens et al. 2001; Suykens and Vandewalle 2002). However, it must be emphasized that global optimization is well known to be nondeterministic

polynomial-time hard (Garey and Johnson 1979), and so it is inevitable that a global optimization method will require substantially more computation to approximate a global solution.

Application Examples

In this section, three applications are presented that illustrate the proposed OpenSees-SNOPT FE-based optimization framework. These applications are: (1) a structural reliability analysis problem; (2) a nonlinear structural optimization problem; and (3) a FE model updating problem. It is noted here that the application examples presented are chosen to highlight the capabilities of the presented OpenSees-SNOPT FE-based optimization framework. Thus, for the sake of clarity and conciseness, these application examples do not treat in detail all the issues encountered in real engineering problems for such applications.

Example 1: FE Reliability Analysis of a Reinforced Concrete Frame Structure

The first application example consists of a FE reliability analysis of a two-dimensional (2D) two-story two-bay reinforced concrete (RC) frame on a rigid foundation, the geometry of which is shown in Fig. 5. This frame structure is modeled using displacement-based Euler-Bernoulli frame elements with distributed plasticity, each with four Gauss-Legendre integration points along its length. Each beam and column of the frame is modeled using three frame elements. Section stress resultants at the integration points along each element are computed through fiber section discretization (Fig. 5). The constitutive behavior of the reinforcement steel is represented via a uniaxial Menegotto-Pinto constitutive model with linear kinematic hardening and zero isotropic hardening (Menegotto and Pinto 1973; Barbato and Conte 2006). The concrete is modeled by using a uniaxial Kent-Scott-Park concrete material model (Scott et al. 1982) with degrading linear unloading/reloading stiffness according to Karsan and Jirsa (1969) and no tensile strength (Barbato et al. 2010a,b). Different material parameters are used for the confined and unconfined concrete in the columns, while the concrete in the beams is modeled as unconfined.

Eleven material constitutive parameters are used to characterize the various structural materials present in the structure; namely,
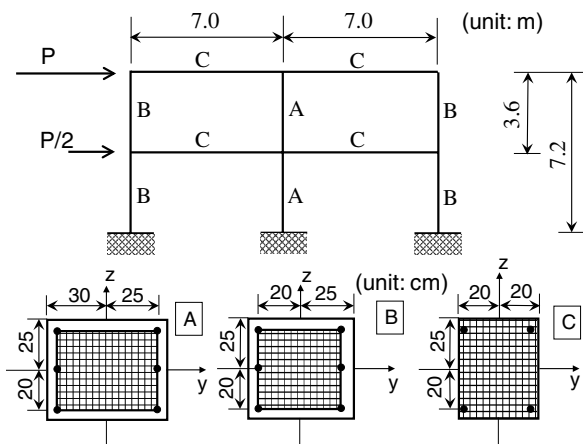
Table 1. Mean, COV, and DP Values of RVs for the Two-Story RC Frame

RV (unit)	Mean	COV	DP
$f_{\text{c,cover}}$ (MPa)	27.59	0.20	25.69
$\epsilon_{\text{c,cover}}$ (−)	2.000×10^{-3}	0.20	1.959×10^{-3}
$\epsilon_{\text{cu,cover}}$ (−)	8.000×10^{-3}	0.20	7.791×10^{-3}
$f_{\text{c,core}}$ (MPa)	34.49	0.20	32.55
$f_{\text{cu,core}}$ (MPa)	20.69	0.20	19.74
$\epsilon_{\text{c,cover}}$ (−)	2.000×10^{-3}	0.20	3.924×10^{-3}
$\epsilon_{\text{cu,cover}}$ (−)	1.400×10^{-2}	0.20	1.367×10^{-2}
f_y (MPa)	248.2	0.20	222.9
E (GPa)	210.0	0.20	209.2
b (−)	2.000×10^{-2}	0.20	1.790×10^{-2}
P (kN)	150.0	0.20	209.2

four parameters each for the confined concrete ($f_{\text{c,core}}$, peak strength; $\epsilon_{c,core}$, strain at peak strength; $f_{\text{cu,core}}$, residual strength; $\epsilon_{\text{cu,core}}$, strain at which the residual strength is reached) and the unconfined (cover) concrete ($f_{\text{c,cover}}$, $\epsilon_{\text{c,cover}}$, $f_{\text{cu,cover}}$, $\epsilon_{\text{cu,cover}}$), and three parameters for the reinforcement steel (E_0 = initial stiffness; f_y = yield strength; b = post yield to initial stiffness ratio). Each material parameter (except $f_{\text{cu,cover}}$, which is assumed to be deterministically equal to zero) is modeled with a single lognormal random variable over the entire structure. The statistical parameters describing the lognormal distribution or each of these material parameters are given in Table 1 and are consistent with studies reported in the literature and based on real data (Mirza and MacGregor 1979; Mirza et al. 1979). The statistical correlation coefficients of the various pairs of material parameters are assumed based on engineering judgment as follows:

- $\rho = 0.7$ for

$$f_{\text{c,core}} = f_{\text{cu,core}}; \quad \epsilon_{\text{c,core}} = \epsilon_{\text{cu,core}}; \quad \epsilon_{\text{c,cover}} = \epsilon_{\text{cu,cover}}$$

$$f_{\text{c,core}} = f_{\text{c,cover}}; \quad \epsilon_{\text{c,core}} = \epsilon_{\text{c,cover}}; \quad \epsilon_{\text{cu,core}} = \epsilon_{\text{cu,cover}}$$

- $\rho = 0.5$ for

$$f_{\text{cu,core}} = f_{\text{c,cover}}; \quad \epsilon_{\text{c,core}} = \epsilon_{\text{cu,cover}}; \quad \epsilon_{\text{cu,core}} = \epsilon_{\text{c,cover}}$$

- $\rho = 0$ for all other pairs of material parameters.

After static application of the gravity loads (defined as uniformly distributed load per unit length of beam $Q = 42.0$ kN/m at each floor), the structure is subjected to a quasi-static monotonic pushover analysis, in which an upper-triangular distribution of horizontal forces is applied at the floor levels (see Fig. 5). The horizontal force, P, applied at the roof level is modeled as a lognormal random variable with mean $\mu_P = 150$ kN and a coefficient of variation (COV) of 20% (see Table 1), while the horizontal force applied at the second floor level is defined as $P/2$ (i.e., fully correlated with the force applied at the roof level). FE response, response sensitivity based on the DDM, and reliability analyses are performed using the FE analysis framework OpenSees, in which the material constitutive models used here were implemented and augmented for response sensitivity analysis using the DDM in previous studies (Barbato and Conte 2006; Gu 2008; Gu et al. 2009a,b).

The numerical optimization problem corresponding to the DP search is described by the following equations:

$$\min_{\mathbf{Y}} F(\mathbf{Y}) = \frac{1}{2}\mathbf{Y}^T\mathbf{Y} \qquad \text{subject to } G(\mathbf{Y}) = u_{\text{lim}} - u_{roof}(\mathbf{Y}) \quad (3)$$

Fig. 5. FE reliability analysis example: geometry, cross-sectional properties, and applied horizontal loads for the two-story two-bay RC frame model

in which u_{roof} = maximum horizontal displacement measured at the top of the middle column of the structural model, and u_{lim} = 0.144 m = limit-state threshold of the horizontal displacement (corresponding to a roof drift ratio of 2.0%). In this problem, the gradients of the objective and constraint (LSF) functions with respect to the random variables $\mathbf{Y}$ are computed by using the DDM as follows:

$$\frac{\partial F(\mathbf{Y})}{\partial \mathbf{Y}} = \mathbf{Y}; \qquad \frac{\partial G(\mathbf{Y})}{\partial \mathbf{Y}} = \frac{\partial u_{roof}}{\partial \mathbf{RV}} \cdot \frac{\partial \mathbf{RV}}{\partial \mathbf{Y}} \qquad (4)$$

in which the gradient $\partial u_{roof}/\partial \mathbf{RV}$ is obtained via the DDM-based response sensitivity analysis, and $\partial \mathbf{RV}/\partial \mathbf{Y}$ = Jacobian matrix of the inverse of the probability transformation between the physical space of random variables $\mathbf{RV}$ and the standard normal space of random variables $\mathbf{Y}$ (Ditlevsen and Madsen 1996).

The DP search is performed twice, by using (1) the newly presented OpenSees-SNOPT framework, and (2) the classical Hasofer-Lind/Rackwitz-Fiessler (HLRF) algorithm (Rackwitz and Fiessler 1978) that already exists in the reliability module of OpenSees. In both cases, the starting point of the DP search is taken as the origin of the standard normal space. The DPs obtained using the two algorithms practically coincide (with differences of the order of the user-defined tolerance), with the first-order reliability index $\beta_{FORM} = 2.094$, and the FORM estimate of the failure probability (or probability of limit-state exceedance) $P_{f,FORM} = \Phi(-\beta_{FORM}) = 0.0181$, where $\Phi(\dots)$ denotes the standard normal cumulative distribution function. The values of the $\mathbf{RV}$s at the DP are reported in Table 1. Fig. 6 plots the total base shear-horizontal floor displacement responses (where u_1 is the horizontal displacement at the first floor and u_{roof} is the horizontal displacement at the roof level, both recorded at the corresponding node of the middle column) of the frame structure subjected to the quasi-static pushover load and with random variables set at their mean values and their DP values, respectively. The reliability results obtained by using FORM are also validated by Monte Carlo simulation (MCS), which yielded the failure probability estimate $P_{f,MCS} = 0.0185$ [from 50,000 realizations of the vector of random variables $\mathbf{RV}$ for COV $(P_{f,MCS}) = 0.0325$], considered here as the reference solution. It is observed that the FORM analysis provides sufficiently accurate estimates of the failure probability for this application example.

In this application example, the performance of the HLRF algorithm is slightly better than that of SNOPT, since it employs nine iterations to find the DP, while the SNOPT-OpenSees framework requires 11 iterations to find the same DP. In addition,
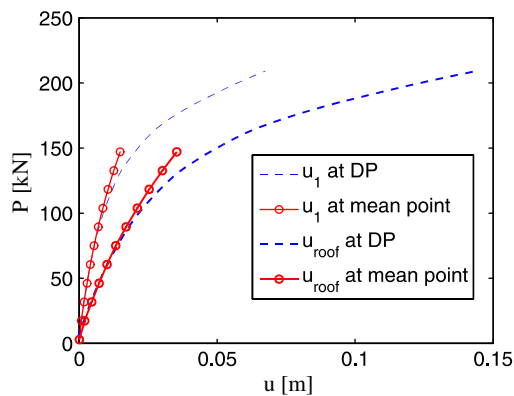
the SNOPT component of the presented framework imposes some additional computational overhead compared with the simpler HRLF algorithm, although this computational overhead is almost negligible compared with the computational cost of a single iteration in the DP search. However, it has been observed by the writers in other applications not reported here because of space constraint that, in general, SNOPT is a more robust DP search algorithm than HLRF. In particular, most of the time when HLRF has convergence difficulties, SNOPT can still successfully obtain the correct DP. For example, this observation applies to the reliability analysis of the structural system considered here, when the limit-state threshold is set to a significantly higher value (e.g., $u_{lim} = 0.216$ m, corresponding to a roof drift ratio of 3.0%). In this case, the difficulty to converge to the DP stems from the fact that, for high limit-state thresholds, the limit-state surface becomes highly nonlinear and nonsmooth, the gradient of the limit-state function also becomes nonsmooth and most likely discontinuous, and the iteration process in the DP search may reach points in the failure domain that are too far from the origin of the standard normal space and for which the FE analysis code may not be able to converge and provide the value of both the limit-state function and its gradient. Similar observations were made by other researchers (Haukaas and Der Kiureghian 2006), who proposed approaches to avoid points deep inside the failure domain. The SNOPT component of the presented FE-based optimization framework already contains remedies to resolve such problems.

Example 2: Structural Optimization of an Electrical Transmission Tower

The second application example consists of the design optimization of a steel electrical transmission tower subjected to wind load. The tower is a five-story three-dimensional (3D) frame structure with bolted joints of height $H = 15$ m and with a square horizontal section, tapering from a side dimension $L_{bottom} = 6$ m at the ground level to a side dimension $L_{top} = 2$ m at the top of the structure (Fig. 7). The tower behaves mainly as a 3D truss structure. Therefore, the structural components are modeled using 3D truss elements, neglecting their flexural behavior (Fig. 7). The steel material is modeled using an asymmetric Menegotto-Pinto constitutive model (Fig. 7), with Young's modulus $E = 210$ GPa, tensile yield strength $f_{yt} = 350$ MPa, compressive yield strength $f_{yc} = 175$ MPa, and strain hardening ratio $b = 0.02$ (i.e., ratio between the postyield stiffness and the Young's modulus). The asymmetry in the material constitutive model is introduced to account for buckling in compression of the truss members. The wind load acting on the tower is modeled as an equivalent static load equally distributed to the four nodes at the top of the structure and inclined by 30° with respect to the x axis (Fig. 7).

The FE-based optimization framework presented in this paper is used to determine the cross-section sizes of the structural members that minimized the weight (and, thus, the cost) of the structure under constraints on the maximum displacement at the top of the structure under different wind loading conditions. Two loading conditions are considered, the first corresponding to serviceability conditions with a total wind load of 100 kN and the second corresponding to ultimate conditions with a total load of 400 kN. The truss members are classified into the following three groups according to their location within the structure and their cross-sectional areas (Fig. 7), which define the three design variables (**DV**s) considered in this problem: (1) the truss members denoted as A defining the columns of the tower, with a common cross-sectional area varying in the range of 8.0×10^{-4} to 1.6×10^{-2} m², and with an initial value of $A_{1,0} = 8.0 \times 10^{-3}$ m² (starting point of the optimization process for design variable A_1); (2) the diagonal truss

Fig. 6. FE reliability analysis example: base shear-horizontal floor displacements of the two-story two-bay RC frame structure
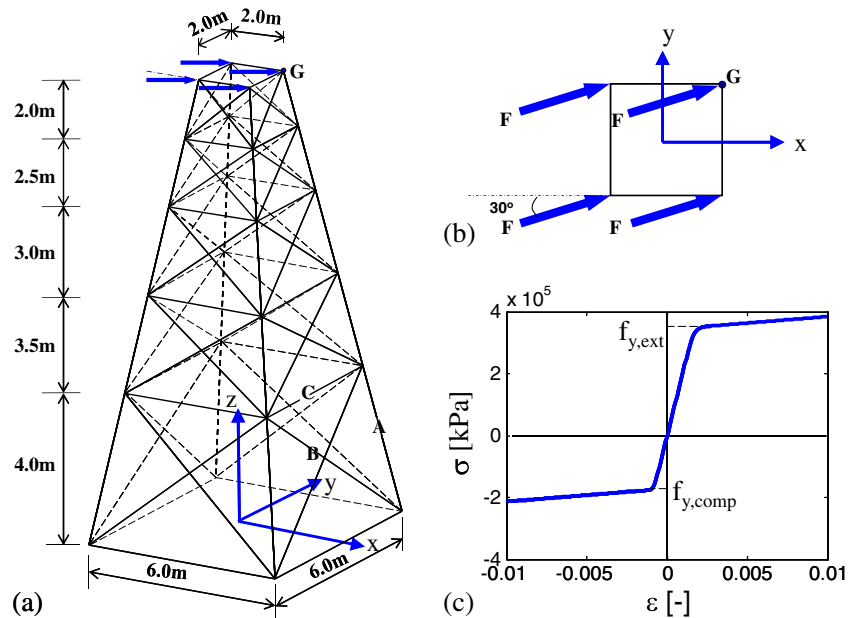
Fig. 7. FE structural optimization example: (a) 3D truss model of electrical transmission tower; (b) loading conditions at the top of tower; (c) asymmetric Menegotto-Pinto constitutive model

members denoted as *B*, with a common cross-sectional area varying in the range of 3.0×10^{-4} to 6.0×10^{-3} m^2, and with an initial value of $A_{2,0} = 3.0 \times 10^{-3}$ m^2 (starting point of the optimization process for design variable A_2); and (3) the horizontal truss members denoted as *C*, with a common cross-sectional area varying in the range of 2.0×10^{-4} to 4.0×10^{-3} m^2, and with an initial value of $A_{3,0} = 2.0 \times 10^{-3}$ m^2 (starting point of the optimization process for design variable A_3). The objective function for the design optimization of this tower is the total volume of steel, while the constraint functions are defined by (1) the upper and lower bounds on the three design variables; (2) an upper bound on the maximum displacement at the top of the structure (measured at Node *G*) under the serviceability loading condition (i.e., $u_{G,s} \leq 0.015$ m); and (3) an upper bound on the maximum displacement at the top of the structure (measured at Node *G*) under the ultimate loading condition (i.e., $u_{G,u} \leq 0.15$ m). Thus, the numerical optimization problem is described by the following equations:

$$\min_{A_1, A_2, A_3} F(A_1, A_2, A_3) = A_1 \cdot \sum_{j=1}^{N_A} L_j + A_2 \cdot \sum_{j=1}^{N_B} L_j + A_3 \cdot \sum_{j=1}^{N_C} L_j$$

subject to $0.0008 \leq A_1 \leq 0.0160$ m^2

$0.0003 \leq A_2 \leq 0.0060$ m^2

$0.0002 \leq A_3 \leq 0.0040$ m^2

$u_{G,s}(A_1, A_2, A_3) \leq 0.015$ m

$u_{G,u}(A_1, A_2, A_3) \leq 0.150$ m   (5)

where N_A, N_B, and N_C = number of truss members of types *A*, *B*, and *C*, respectively, and L_j = length of truss member *j*. In this case, the gradients of the objective and constraint functions are computed using both the DDM and the finite-difference method. Because of space constraints, only the results corresponding to the use of the finite-difference method are presented here. It is noteworthy that the final results, in terms of the number of iterations and values of the design variables at the optimum design, are practically coincident for the two cases. At each iteration of the optimization process, gravity is applied first in one load step and then the

wind load is applied incrementally with Newton-Raphson iterations at each step. A total of 40 iterations and less than 10 s (on an ordinary personal computer with CPU frequency = 2.4 GHz) are required to reach the optimum design, which is found as: (1) $A_1 = 3.17 \times 10^{-3}$ m^2, (2) $A_2 = 3.51 \times 10^{-4}$ m^2, and (3) $A_3 = 2.0 \times 10^{-4}$ m^2. The optimal value of variable A_3 coincides with its lower bound, which showed that most of the wind forces are resisted by truss members *A* and *B*, and truss member *C* is relatively ineffective for this loading. However, truss member *C* is important in supporting/confining the truss subsystems and avoiding global stability problems (e.g., global buckling). The response curve corresponding to the total applied wind force versus total displacement of Node G is shown in Fig. 8 for (1) the initial design (with all **DV**s set at their initial values); (2) an intermediate iteration of the optimization process (Iteration No. 20); and (3) the converged optimal design (corresponding to Iteration No. 40). It is observed that, at the optimal design, the truss tower yields significantly under the ultimate loading condition. The inset of Fig. 8 shows the volume of steel needed to build the tower as a function of the number of iterations in the optimization
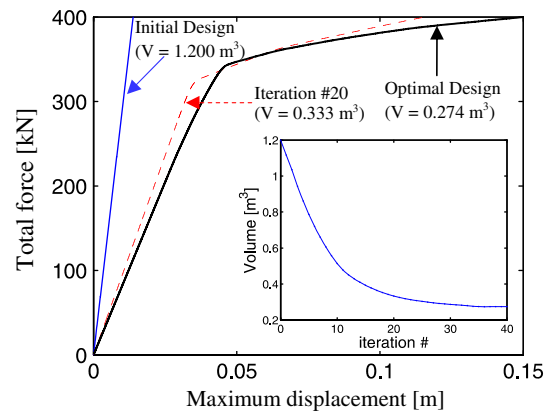

Fig. 8. FE structural optimization example: total applied wind force versus total displacement response of Node G (see Fig. 7)

process. The volume of steel required is 1.200 m^3 for the initial design and 0.274 m^3 at the optimal design. Thus, in this case, the optimization process produces a savings of 77% of the material in volume or weight. It is noteworthy that the total displacement of Node G for the optimal design is $u_{G,s} = 0.012$ cm under the serviceability loading condition (i.e., a total wind load of 100 kN) and $u_{G,u} = 0.150$ cm under the ultimate loading condition (i.e., a total wind load of 400 kN). Thus, in this specific case, the constraint corresponding to the maximum total displacement at the ultimate loading condition controls the optimal design; i.e., when the constraint for the ultimate loading condition is exactly satisfied, the constraint corresponding to the maximum total displacement at the serviceability loading condition is more than satisfied.

Example 3: Nonlinear FE Model Updating of a 2D Clay Soil Column

The FE method is commonly used in the design and analysis of structural and geotechnical systems. Because of (1) incomplete knowledge or statistical uncertainty of the material, geometric, and other mechanical/mechanistic parameters characterizing the structural/geotechnical systems, and (2) simplified modeling assumptions based on engineering judgment and intuition, a FE model based on the initial estimates of these parameters can be unable to represent the behavior of the actual system with the level of accuracy required by a specific application. The objective of FE model updating is to correct the initial FE model to tune it to available experimental data (e.g., measured response to actual earthquake records or identified mode shapes and frequencies), in order to obtain an updated FE model that can represent with higher fidelity the mechanical behavior of the considered structural/ geotechnical system. Two general classes of methods are used in FE model updating, i.e., direct methods and sensitivity-based methods. Direct methods update the global stiffness and/or mass matrices that appear in the structural equations of motion of the system under study (Baruch 1982; Wei 1990). The sensitivity-based methods adjust a preselected set of physical parameters to minimize an objective function measuring the discrepancy between the computed and measured structural responses.
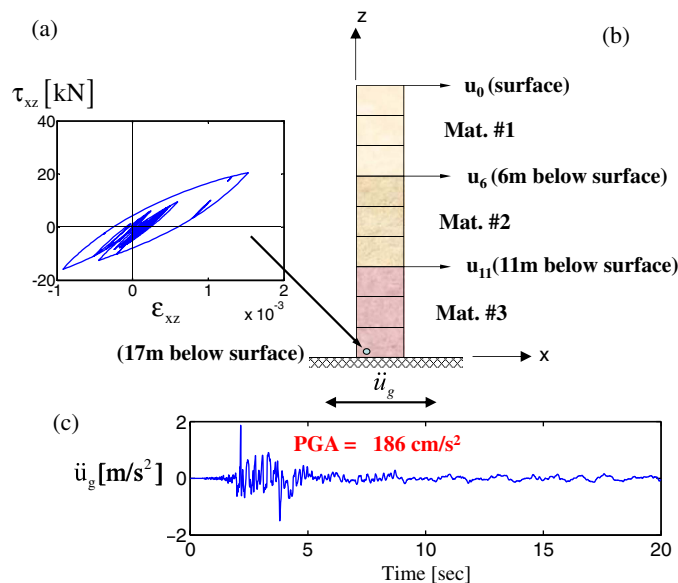
In this application example, a sensitivity-based method is used for the nonlinear FE model updating of a 2D clay soil column extending from the ground surface to a depth of 17 m [Fig. 9(b)]. Nine 2D four-node quadratic elements are used to model the soil column assumed under plane strain condition in the (x, z) plane. The FE model of the soil column has fixed boundary conditions at the bottom, while the two nodes of each pair of nodes of the model at the same depth on the two lateral boundaries are tied together (i.e., their horizontal and vertical displacements are constrained to be the same) to produce a shear beam model. The soil materials are modeled using a 3D pressure-independent J_2 multiyield surface clay model (Elgamal et al. 2003) specialized for 2D plane strain conditions. The soil column is subdivided into three layers characterized by different material properties defined by the low strain shear modulus G_i and shear strength τ_i for each layer of the soil ($i = 1, 2, 3$). The top soil layer, extending from the ground surface to a depth of 6 m, is identified in Fig. 9(b) by the Mat. #1 label; the intermediate soil layer, extending from 6 to 11 m below the ground surface, is identified in Fig. 9(b) by the Mat. #2 label; and the bottom soil layer, extending from a depth of 11 to 17 m, is identified in Fig. 9(b) by the Mat. #3 label. The material properties corresponding to each of the three soil layers are given in Table 2. Zero viscous damping is assumed and only material damping, deriving from the inelastic material behavior, is considered in the FE model of the soil column.

The soil column is subjected to a base excitation taken as an actual downhole acceleration record measured at a depth of 17 m below the ground surface, at a site near Lotung, China, during the earthquake of July 30, 1986, with a peak ground acceleration of 1.86m/s^2 (Station Code DHB17, No. 12, oriented in the north-south direction) (Elgamal 1996). The time history of the earthquake excitation is shown in Fig. 9(c). First, OpenSees is used to perform a FE dynamic analysis for base excitation to obtain the reference seismic responses of the soil column, corresponding to the aforementioned reference nonlinear FE model of the soil column. Three monitoring stations are considered, located at the ground surface and at depths of 6 and 11 m, and their acceleration histories are recorded as computed by OpenSees. The stress-strain response computed near the base of the soil column [shown in Fig. 9(a)] shows that the soil yields significantly during the earthquake.

To test the presented SNOPT-OpenSees optimization framework for FE model updating, the reference FE model of the soil column is purposely modified by changing the values of the soil material parameters and obtaining the initial FE model to be updated. The values of the soil material parameters of this initial FE model, assumed to be the starting point of the FE model updating process, are given in Table 2. The FE model updating problem is defined as a least-squares problem (Link 1999), in which the difference between the time histories of the accelerations recorded (here in the reference FE model) and predicted by the updated

Fig. 9. FE model updating example: (a) soil shear stress-shear strain response; (b) FE model of the soil column; (c) base acceleration time history

Table 2. Soil Material Parameter Values Used in the Nonlinear FE Model Updating of a Soil Column

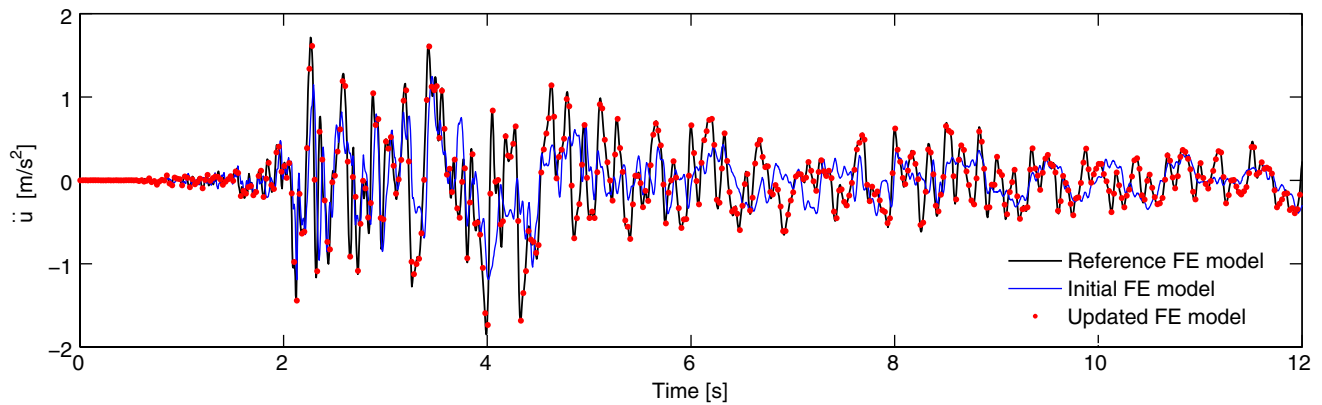| Parameter (kPa)	Reference model	Initial FE model to be updated	SNOPT FE model updating result
G_1	28800.0	30000.0	28800.0
G_2	39200.0	30000.0	39199.9
G_3	57800.0	30000.0	57800.1
τ_1	31.0	30.0	31.0
τ_2	33.0	30.0	33.0
τ_3	34.0	30.0	34.0

Fig. 10. FE model updating example: comparison of soil acceleration response histories at ground surface from (1) reference FE model, (2) initial FE model, and (3) updated FE model

FE model at the three monitored stations is minimized in the least-squares sense, subjected to constraints on the values of the optimization parameters. The mathematical form of the optimization problem is as follows:

$$\min_{\mathbf{DV}} F(\mathbf{DV}) = \frac{1}{2} \sum_{j=1}^{3} \left\{ \sum_{n=1}^{N_t} [\ddot{u}_j(t_n, \mathbf{DV}) - \ddot{u}_j^{ref}(t_n)]^2 \right\}$$

subject to $\quad G_i \geq 20{,}000 \text{ kPa}, \quad i = 1, 2, 3$

$$\tau_i \geq 20 \text{ kPa}, \quad i = 1, 2, 3 \qquad (6)$$

where $\mathbf{DV} = [G_1, G_2, G_3, \tau_1, \tau_2, \tau_3]^T$ = vector of optimization parameters; superscript T = transposition operator; N_t = number of time steps in the dynamic FE analysis; t_n = discrete time at the end of the nth time step; $\ddot{u}_j^{ref}(t_n)$ = value of the ground acceleration recorded at time t_n at the jth monitored station in the reference FE model; and $\ddot{u}_j(t_n, \mathbf{DV})$ = value of the ground acceleration computed at time t_n at the jth monitored station in the FE model to be updated (i.e., with optimization parameters assuming the values $\mathbf{DV}$). Also, in this application example, the gradient of the objective function is computed using the DDM as

$$\frac{\partial F(\mathbf{DV})}{\partial \mathbf{DV}} = \sum_{j=1}^{3} \left\{ \sum_{n=1}^{N_t} [\ddot{u}_j(t_n, \mathbf{DV}) - \ddot{u}_j^{ref}(t_n)] \cdot \frac{\partial \ddot{u}_j(t_n, \mathbf{DV})}{\partial \mathbf{DV}} \right\} \quad (7)$$

where $\partial \ddot{u}_j(t_n, \mathbf{DV})/\partial \mathbf{DV}$ = sensitivities of the FE response $\ddot{u}_j(t_n, \mathbf{DV})$ with respect to the optimization parameters $\mathbf{DV}$. The results of the nonlinear FE model updating analysis are reported in Table 2. It is observed that the SNOPT-OpenSees optimization framework is able to very accurately recover the values of the optimization parameters corresponding to the reference FE model. Fig. 10 compares the first 12.0 s of the soil acceleration response histories recorded at the ground surface in the reference FE model, initial FE model to be updated, and final FE model after FE model updating. As expected, it is observed that while the difference between the acceleration time histories from the reference FE model and the initial FE model is significant, the acceleration time histories from the reference FE model and the updated FE model practically coincide.

Conclusions

This paper presents a general FE-based optimization framework obtained by integrating two powerful computer programs: Open-Sees, an object-oriented FE software framework for simulating the response of structural and geotechnical systems to static and dynamic loads, with special emphasis on earthquake loading; and SNOPT, a software package for solving large-scale unconstrained and constrained optimization problems. The major features of this framework are (1) its advanced capabilities in solving complex civil engineering optimization problems; (2) its computational efficiency; (3) its versatility in solving a wide range of structural and geotechnical problems; (4) its flexibility in readily incorporating new developments in computational structural and soil mechanics, nonlinear FE analysis solution strategies, numerical optimization, and probabilistic modeling and analysis; and (5) its capability of being easily extended to new problems and analysis types. Feature No. 5 is obtained by allowing users to write their own objective and constraint functions, making the application possibilities of the OpenSees-SNOPT framework practically unlimited. The capabilities of this new extended framework are illustrated through three numerical application examples: (1) the FE reliability analysis of a reinforced concrete frame structure, (2) the structural optimization of an electrical transmission tower, and (3) the nonlinear FE model updating of a soil column or profile.

In conclusion, the presented OpenSees-SNOPT optimization framework provides a powerful tool for FE-based optimization problems in civil structural and geotechnical engineering. It is noteworthy that the software architecture of the integration of SNOPT and OpenSees presented in this paper is quite general and can be used to integrate any optimization software with any object-oriented FE analysis software other than OpenSees.

writers and do not necessarily reflect the views of the sponsoring agencies.

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