Probabilistic Nonlinear Response Analysis of Steel-Concrete Composite Beams

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Abstract: This paper employs a methodology for probabilistic response analysis based on the first-order second moment (FOSM) method in conjunction with response sensitivity computation through the direct differentiation method (DDM), to study the variability of the structural response of steel-concrete composite (SCC) beams. This methodology is applied to compute the first-order and second-order statistical moments of the response of two actual structural systems for which experimental data are available. The results of the DDM-based FOSM method are compared with the experimental measurements and with the results of the computationally more expensive Monte Carlo-Simulation (MCS) method. Different modeling hypotheses for the material parameter uncertainty are considered. The DDM-based FOSM method agrees very well with the MCS results for low-to-moderate levels of response nonlinearity under low-to-moderate material parameter uncertainty and up to high level of response nonlinearity under low material parameter uncertainty. The DDM-based FOSM method is shown to correctly describe the effects of random spatial variability of material parameters. **DOI: 10.1061/(ASCE)ST.1943-541X.0000803.**

Author keywords: Probabilistic response analysis; Finite-element method; Nonlinear material constitutive models; Steel-concrete composite structures; Deformable shear connection; Analysis and computation.

Introduction

In the last decades, steel-concrete composite (SCC) structures have been widely used in building and bridge construction, motivating several studies devoted to modeling, analysis, and design issues (e.g., Viest et al. 1997; Oehlers and Bradford 1999; Galambos 2000; Mazzolani 2003; Spacone and El-Tawil 2004). There is significant interest in the evaluation and practical application of methods to propagate uncertainty from the parameters defining the model of a structure (loading conditions, material and geometric properties, and other structural parameters having significant random variability) to the engineering demand parameters (EDPs), as needed for safety assessment, optimum design of new structures, and optimum retrofit/maintenance of existing structures. Recent studies investigated the response sensitivity of finite-element (FE) models of SCC structures to their material constitutive parameters (Zona et al. 2005, 2006; Barbato et al. 2007) and the effects of model parameter uncertainty on the EDPs of SCC structures (Amadio 2008; Zona et al. 2010). Significant research has been devoted to methods for propagating uncertainty from model parameters to EDPs through FE analysis (Der Kiureghian and Ke 1988), with studies focusing on two complementary perspectives, namely reliability analysis (Ditlevsen and Madsen 1996) and probabilistic response analysis (Grigoriu 2000). Reliability analysis focuses on rare and/or extreme events (e.g., structural collapse, exceedance of high thresholds by EDPs, loss of serviceability) and thus, is primarily concerned with the accurate estimation of the tail of the probability density functions (PDFs) of the EDPs. On the other hand, probabilistic response analysis focuses on the low-order statistical moments characterizing the body of the PDFs of the EDPs. Probabilistic response analysis involves computing the probabilistic characterization (i.e., statistical moments) of structural response parameters, given as input the probabilistic characterization of material, geometric, and loading parameters. Existing analytical and semianalytical probabilistic response analysis methods are limited to linear structural models (Madsen and Bazant 1983; Bazant and Liu 1985; Katafygiotis and Beck 1995) or simple nonlinear structural models (Soize 1995). Numerical probabilistic response analysis methodologies based on the FE method are able to provide approximate probabilistic information about EDPs from state-of-the-art mechanics-based models of real-world structures (Lawrence 1987; Ghanem and Spanos 1991; To 2001; Schueller 2001; Noh 2004; Stefanou 2009). However, these numerical techniques are usually computationally expensive and require a compromise between accuracy and computational effort. Therefore, simplified probabilistic response analysis methods, based on the combination of analytical/semianalytical probabilistic methods and the FE method, and sufficiently accurate for engineering purposes, are of particular practical and theoretical interest. On one hand, the efficient integration of advanced nonlinear FE analysis and simplified probabilistic response analysis is an appealing approach to quantify the response variability of complex large-scale systems by using tools that are well-known and widely accepted by the design community. On the other hand, this integration can be beneficial in the area of performance-based analysis and design of SCC structures, as well as in the calibration of the partial resistance factors needed in modern design codes.

This study analyzes the effects on the structural response of the uncertainties in material and shear connection constitutive

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Note. This manuscript was submitted on January 18, 2012; approved on January 14, 2013; published online on January 16, 2013. Discussion period open until February 23, 2014; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Structural Engineering***, © ASCE, ISSN 0733-9445/04013034(10)/$25.00.

parameters used to define the FE models of SCC beams. It is assumed that the statistical information available is limited to the first-order and second-order statistics (means, variances, and correlation coefficients) of the material constitutive parameters, and the statistical information of interest consists of first-order and second-order statistics of structural response parameters, also referred to as EDPs. This investigation is performed using two different probabilistic response analysis methods, based on nonlinear static FE response analysis of SCC structures and characterized by different accuracy and computational cost, i.e., Monte Carlo simulation (MCS) and the first-order second-moment (FOSM) method (Ang and Tang 1975; Wong 1985). The FOSM method is used in conjunction with response sensitivity computation through the direct differentiation method (DDM), resulting in the DDM-based FOSM method (Haukaas and Der Kiureghian 2005; Barbato et al. 2010). Probabilistic response analysis results are presented in detail for a simply supported beam and a two-span continuous beam, for which experimental test results and statistical data on the material model parameters are available.

The contributions of this paper consist in: (1) applying the DDM-based FOSM method as a computationally efficient method to propagate uncertainty from model parameters to EDPs of SCC structures through advanced nonlinear FE analysis by using experimentally validated FE models; (2) evaluating the advantages and limitations of the FOSM method when compared with MCS, as applied to real-world nonlinear SCC beam structures for which experimental data are available; and (3) investigating the effects of model parameter uncertainty on the predicted response of SCC beams, with particular emphasis on the comparison of the results obtained by estimating the coefficient of variation (COV) of the material parameters from material tests specifically performed for the subject structure and by using typical values of these COVs recommended in the literature. It is noteworthy that SCC structures present similar issues in FE modeling and probabilistic characterization of material parameters as both (reinforced concrete) RC and steel structures, but also very specific deterministic and probabilistic modeling issues related to the interaction between the RC and steel components. Thus, probabilistic response analysis methods developed for SCC structures are also suitable for RC-only or steel-only structures as special cases.

Review of Finite-Element-Based Probabilistic Response Analysis

This study considers two FE-based probabilistic response analysis methods, i.e., crude MCS (Liu 2001), and the mean-centered DDM-based FOSM method (Haukaas and Der Kiureghian 2005; Barbato et al. 2010). The relevance of FE-based probabilistic response analysis methods derives from the fact that, for real-world problems, FE analysis is widely used to perform response simulation (i.e., computation of response quantities $\mathbf{r} = [r_1, r_2, \ldots, r_n]^T$ for given values of a set of random parameters $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_m]^T$, in which the superscript T denotes the vector/matrix transposition operator). This study focuses on probabilistic response analysis methods based on quasi-static nonlinear FE analysis used to simulate the response of structural component/systems subjected to quasi-static experimental testing.

Probabilistic Response Analysis through Monte Carlo Simulation

Crude MCS is a general and accurate but computationally expensive probabilistic response analysis method. MCS requires knowledge of the joint PDF of the random parameters, $\boldsymbol{\Theta}$, which is, in general, only partially known. Thus, appropriate probability distribution models, consistent with the incomplete statistical information available, must be used to generate realizations of the vector $\boldsymbol{\Theta}$. In addition, the number, N, of FE simulations required by MCS can be large for accurate estimation of marginal and joint moments of response quantities, $\mathbf{R}$, and increases rapidly with the order of the moments. Because each simulation, for real-world structures, may involve a complex nonlinear FE analysis, repeating such analyses a large number of times could be computationally prohibitive.

In this study, MCS is used as a reference solution for the significantly less computationally intensive FOSM method. In the MCS analysis, the FE models are built using the various realizations of the model parameters, simulated appropriately from a Nataf model (for the joint PDF) consistent with the available (incomplete) statistical information (Melchers 1999).

Probabilistic Response Analysis through First-Order Second-Moment Method

The FOSM method is a simplified and computationally inexpensive method for FE-based probabilistic response analysis. It estimates the mean values (first-order statistical moments), variances, and covariances (second-order statistical moments) of the FE response parameters or EDPs of interest by using a first-order Taylor series expansion of these nonlinear response quantities in terms of the random/uncertain model parameters (Ang and Tang 1975; Wong 1985). In this study, only the mean-centered (i.e., Taylor series expansion about the mean point) DDM-based FOSM method is employed and referred to as FOSM analysis. The FOSM method requires only the knowledge of the first-order and second-order statistical moments of the random parameters. Often statistical information about the random parameters is limited to first and second moments. Thus, in such cases, probabilistic response analysis methods more advanced than FOSM analysis cannot be fully exploited.

In the sequel, uppercase letters $\boldsymbol{\Theta}$, Θ, $\mathbf{R}$, and R denote random quantities, the corresponding lower case letters $\boldsymbol{\theta}$, θ, $\mathbf{r}$, and r denote specific realizations of these random quantities, bold fonts are used to denote matrix/vector quantities, and regular fonts are employed for scalar quantities. Given the vector $\boldsymbol{\Theta}$ of m random parameters, with covariance matrix $[\boldsymbol{\Sigma}_{\boldsymbol{\theta}}]_{ij} = \rho_{ij} \cdot \sigma_i \cdot \sigma_j;\ i,\ j = 1, 2, \ldots, m$, where ρ_{ij} = correlation coefficient of random parameters Θ_i and Θ_j ($\rho_{ij} = 1$; $i = 1, 2, \ldots, m$), and σ_i = standard deviation of random parameter Θ_i, the vector $\mathbf{R}$ of the n response quantities of interest is approximated by a first-order truncation of its Taylor series expansion in the random parameters $\boldsymbol{\Theta}$ about the mean vector, $\boldsymbol{\mu}_{\boldsymbol{\Theta}}$, of the parameter values (Ang and Tang 1975; Wong 1985). The first-order and second-order statistical moments of the response quantities $\mathbf{R}$ are approximated by the corresponding moments of the linearized response quantities as follows:

$$\boldsymbol{\mu}_{\mathbf{R}} \approx \mathbf{r}(\boldsymbol{\mu}_{\boldsymbol{\Theta}}) \tag{1}$$

$$\boldsymbol{\Sigma}_{\mathbf{R}} \approx \nabla_{\boldsymbol{\theta}}\mathbf{r}|_{\boldsymbol{\theta}=\boldsymbol{\mu}_{\boldsymbol{\Theta}}} \cdot \boldsymbol{\Sigma}_{\boldsymbol{\Theta}} \cdot (\nabla_{\boldsymbol{\theta}}\mathbf{r}|_{\boldsymbol{\theta}=\boldsymbol{\mu}_{\boldsymbol{\Theta}}})^T \tag{2}$$

in which $[\nabla_{\boldsymbol{\theta}}\mathbf{r}]_{ij} = \partial r_i/\partial\theta_j$ ($i = 1, \ldots, n$ and $j = 1, \ldots, m$). Eqs. (1) and (2) show that the FOSM approximation does not require information about the marginal and joint PDFs of the random parameters, which implies that using different probability distributions of the model parameters, for a given mean vector $\boldsymbol{\mu}_{\boldsymbol{\Theta}}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\Theta}}$, leads to the same FOSM estimates of the means, variances, and covariances of the response parameters. However, the FOSM estimate of the covariance matrix of the response parameters depends on the discretization of the random fields describing the spatial variability of the model parameters.

The approximate first-order and second-order response statistics computed using Eqs. (1) and (2) provide information on the variability of the EDPs and on their statistical correlation. They can also be used to evaluate the relative importance of the random parameters in terms of their probabilistic influence on the EDPs (Haukaas and Der Kiureghian 2005; Lee and Mosalam 2005; Barbato et al. 2010). These response statistics can be readily obtained when response sensitivities (i.e., components of the gradient $\nabla_{\boldsymbol{\theta}}\mathbf{r}$ of the response quantities $\mathbf{r}$ with respect to parameters $\boldsymbol{\theta}$) evaluated at the mean values of the random parameters are available. In fact, in addition to FE response-only computation, FOSM analysis requires FE response sensitivity calculations (Kleiber et al. 1997), which are performed in this study through the DDM (Vidal et al. 1991; Zhang and Der Kiureghian 1993; Kleiber et al. 1997; Conte et al. 2003). The DDM is an accurate FE response sensitivity analysis method, particularly efficient for structural models characterized by nonlinear hysteretic behavior. The DDM consists of differentiating analytically the space- and time-discretized equations of equilibrium/motion for the FE model of the structure considered. Notably, only a single FE analysis, with all model parameters set at their mean value, is needed to perform a DDM-based FOSM probabilistic response analysis, leading to a significant decrease in computational cost compared to MCS.

Probabilistic Finite-Element Modeling of Steel-Concrete Composite Beams

Finite-Element Formulation for Response and Response Sensitivity Analysis

In this study, the SCC beams are modeled using composite frame FEs with deformable shear connection. A composite frame FE with deformable shear connection has important advantages over the ordinary Euler-Bernoulli monolithic frame element, namely: (1) more accurate modeling of the structural mechanical behavior including partial composite action; (2) description of the slab-beam interface slip and shear force behavior; (3) evaluation of the effects of the interface slip on the stress distribution; and (4) inclusion of damage and failure of the connectors. The two-dimensional composite beam formulation used in this study is based on the Newmark et al. (1951) model [Fig. 1(a)]. In this model, Euler-Bernoulli beam theory (in small deformations) applies to both components of the composite beam, and the deformable shear connection is represented by an interface model with distributed bond, allowing interlayer slip and enforcing contact between the steel and concrete components. A simple and effective two-dimensional 10-degree-of-freedom (DOFs) displacement-based SCC frame element with deformable shear connection (Dall'Asta and Zona 2002) is employed [Fig. 1(b)]. This locking-free element (Dall'Asta and Zona 2004b) was validated through comparison of numerical simulations and experimental results for monotonic (Dall'Asta and Zona 2004a) and cyclic loads (Zona et al. 2008). Its formulation was augmented for DDM-based response sensitivity analysis, which was then validated by comparison with finite difference calculations of FE response sensitivities (Zona et al. 2005, 2006). Other alternative FE formulations were shown to also provide good predictions of experimental results, e.g., two-field mixed FE (Ayoub and Filippou 2000), force-based FE (Salari and Spacone 2001), three-field mixed FE (Dall'Asta and Zona 2004c), strain-assumed FE (Cas et al. 2004). Among these alternative formulations, only the three-field mixed FE was augmented for

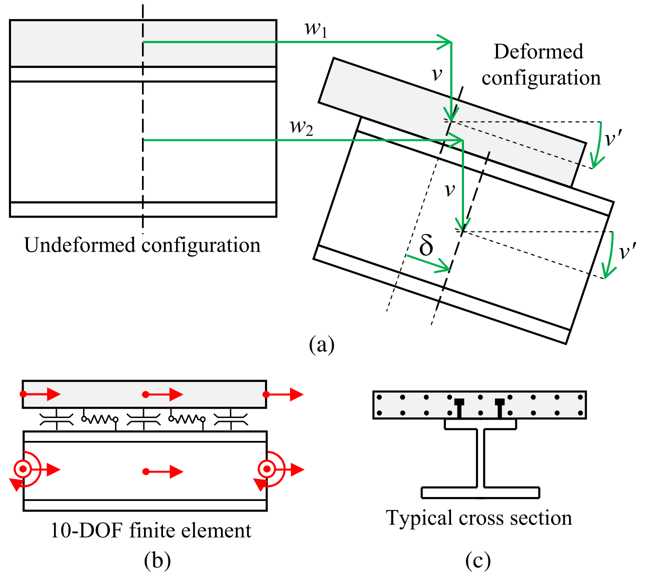

Fig. 1. Composite beam model: (a) kinematic model; (b) finite-element with degrees of freedom; (c) typical cross section

DDM-based response sensitivity (Barbato et al. 2007). However, the displacement-based FE was preferred in this study for its higher robustness compared with the three-field mixed FE (Dall'Asta and Zona 2004c).

In this study, first-order and second-order statistical moments of the material parameters are specified from material test data, when available, or by using statistical/probabilistic information obtained from the literature. Typically, data are insufficient to reliably determine appropriate probability distributions for all material parameters. Therefore, in such cases, different hypotheses on probability distributions of material parameters need to be considered.

Deterministic and Probabilistic Modeling of Constructional Steel Material

The material stress-strain behavior of steel beams is represented in this paper through the uniaxial Menegotto and Pinto (1973) constitutive model, which was extended for FE response sensitivity computation in Barbato and Conte (2006). The three material parameters of this model are: the yield stress, f_{y0}, Young's modulus, E_0, and the strain hardening ratio, b (i.e., the ratio between post-yield and elastic stiffness). When specific experimental data are not available, information on the variability of the material properties of constructional steel can be obtained from several studies reported in the literature, e.g., see Melchers (1999) for a review of earlier data, and Simões da Silva et al. (2009) for more recent results.

The variability of the yield stress is influenced by a number of parameters, e.g., steel grade, thickness of the test samples, and the customary habit of classifying rejected higher grade steel as the next lower grade. Previous studies (Melchers 1999; Simões da Silva et al. 2009) indicate that the yield stress has a COV in the range 0.08–0.13, depending on the steel productions considered. Various probability distributions have been proposed, namely lognormal, normal, and beta. Regarding the Young's modulus, experimental tests show very little scatter, with a COV lower than 0.06 regardless of the steel grade, whereas an appropriate probability distribution is not clearly identified from the literature. Regarding the strain hardening ratio, different values have been suggested in tension and in compression, with a common COV $= 0.25$ (Melchers 1999), whereas an appropriate probability distribution cannot be identified from the literature. For the Young's modulus and strain hardening ratio, both normal and lognormal distributions have been used in the literature. The three material parameters (f_{y0}, E_0, b) of constructional steel are assumed uncorrelated, i.e., with correlation coefficients $\rho_{f_{y0},E_0} = \rho_{f_{y0},b} = \rho_{E_0,b} = 0$.

Deterministic and Probabilistic Modeling of Concrete and Reinforcement Steel Materials

The selected constitutive law for the concrete material in compression is a uniaxial cyclic law with monotonic envelope defined by the Popovics-Saenz law (Balan et al. 2001). The constitutive material parameters modeled as random variables are the initial tangent stiffness (Young's modulus), E_c, the compressive strength, f_c, and the corresponding strain, ε_p, the softening stress, f_0, at the inflection point of the softening branch, and the corresponding strain, ε_0. When specific experimental data are not available, the COVs for E_c, f_c, and ε_p are typically taken as 0.20, as suggested by studies reported in the literature (Mirza et al. 1979). Similarly, the COVs for f_0 and ε_0, for which no data are available experimentally or in the literature, are assumed equal to 0.20. Based on engineering judgment, the statistical correlation coefficients between f_c and f_0, and between ε_p and ε_0 are assumed equal to 0.8, to reduce to negligible values the probability of physically unrealizable combinations of material parameter values (i.e., $\varepsilon_0 < \varepsilon_p$ and/or $f_c < f_0$) in the definition of the relevant concrete constitutive laws. The statistical correlation coefficients for all other pairs of material constitutive parameters are taken as zero.

The reinforcement steel is modeled using a Menegotto and Pinto model (1973), with statistical properties of the model parameters obtained from experimental data or, when data are not available, from Mirza and MacGregor (1979), who report COVs equal to 0.033 and 0.106 for E_0 and f_{y0}, respectively. All reinforcement steel parameters are assumed to be statistically independent and thus uncorrelated.

Deterministic and Probabilistic Modeling of Shear Connection

Various types of shear connection are available for SCC beams, with welded headed shear studs being the most common in construction today (Viest et al. 1997; Oehlers and Bradford 1999). The behavior of a stud connector depends on the stud details (height, diameter, and strength), as well as on the concrete properties, slab detailing (e.g., solid slab, slab with profiled steel sheeting, hollow-cored slab), and reinforcement detailing. Many studies on the static behavior of stud shear connectors are available in the technical literature, from experimental work that established constitutive laws and formulas to estimate the connector bearing capacity (e.g., Ollgaard et al. 1971), to experimental and numerical studies of stud behavior under quasi-static monotonic loading (e.g., Oehlers and Johnson 1987; An and Cederwall 1996; Lam and El-Lobody 2005; Xue et al. 2008) and cyclic loads (e.g., Gattesco and Giuriani 1996; Gattesco et al. 1997; Bursi and Gramola 1999; Civjan and Singh 2003). The cyclic constitutive law adopted in this study for the shear connection is described in Zona et al. (2008). The parameters needed to define the constitutive behavior of the shear connection are the connection strength, $f_{s,\max}$, the ultimate slip, δ_{ult}, and the residual friction, τ_{fr}. Fig. 2 shows a typical cyclic shear force-slip response obtained using the adopted constitutive model. Attributable to the influence of many detailing parameters, the experimental data available do not allow to identify a probability distribution model for the constitutive parameters used to describe the force-deformation behavior of the shear connection. In this

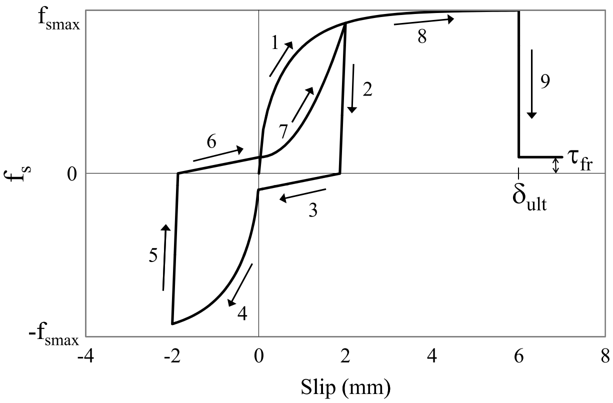

Fig. 2. Constitutive model for the shear connection

study, a COV of 0.20 is assumed for the shear connection strength, i.e., similar to the COV of the concrete material parameters, assuming that the shear connection failure due to concrete crushing is the foremost source of uncertainty. This value of the COV is consistent with the COV values adopted in reliability analysis studies found in the literature (Ubejd Mujagi¢ and Easterling 2009). No statistical information was found in the literature for the ultimate slip, δ_{ult}, and the residual friction, τ_{fr}, which are modeled in this paper as deterministic variables with the following values: $\delta_{\text{ult}} = 6$ mm and $\tau_{fr} = 0$ kN/m. The shear connection strength is assumed uncorrelated to the parameters of the other materials.

Modeling of Statistical Spatial Correlation

The considered material parameters need to be modeled as random fields to rigorously represent their random spatial variability (i.e., location-to-location variability) (Stefanou 2009). However, experimental data are typically insufficient to determine the correlation structure (auto-correlation and cross-correlation functions) for all material parameters. In this study, a less rigorous but more practical approach is adopted to study the effects of spatial correlation by considering the following two extreme hypotheses: (1) a single random variable over the entire structure is used for each material parameter (assumption corresponding to random fields with correlation lengths much larger than the length of the beam structure analyzed, which are equivalent to infinite correlation lengths for all practical purposes, and no spatial variability within the structure); and (2) the uncertainty of each material parameter is described using a set of uncorrelated random variables, each corresponding to one of the FEs used to discretize the structure (assumption corresponding to random fields with correlation lengths much smaller than the length of the shortest FE used to discretize the beam structure and, thus, significant spatial variability within the structure). Noteably, a proper random field discretization method should be used if sufficient experimental data are available to build a realistic model of spatial correlation. However, the intent of investigating the effect on the response variability of the random spatial variability of all the material model parameters considered is already achieved in the present case by using this simplified two extreme cases approach.

Application Examples

Simply Supported Beam

The first benchmark problem considered in this study consists of a 5.00-m-long simply supported beam [Fig. 3(a)], tested by

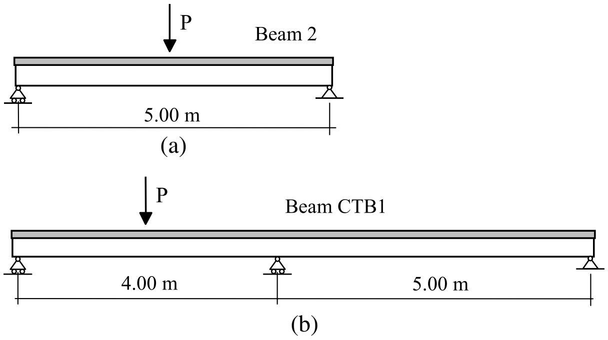

Fig. 3. Beams used as benchmark applications: (a) simply supported beam; (b) continuous beam (data from Ansourian 1981, 1982)

Ansourian (1982) under a vertical concentrated monotonic quasi-static load P applied at midspan, and referred to as Beam 2. The joist section is a European IPBL200, and the RC slab section dimensions are $104 \times 1{,}000$ mm^2. Because of the relatively narrow width of the concrete slab, shear lag effects are neglected in the modeling of this specimen. The reader is referred to Ansourian (1982) for all details regarding the geometry and material properties. Additional unpublished data on the material testing (including the results obtained from the individual material tests) were provided to the writers by Professor Ansourian (personal communication, 2007) and were used to estimate the mean and COV of the concrete compressive strength f_c and of the lower yield stress (also known as static yield stress) (see Bruneau et al. 1998) f_{y0} of the joist steel (Table 1). From a preliminary deterministic sensitivity analysis performed using the DDM, it was found that these two model parameters (f_c and f_{y0}) are the most influential material parameters on the response of the benchmark structure considered in this study. For all other material parameters, the mean values were obtained from the test data provided in Ansourian (1982), and typical values suggested or used in the literature were assumed for their COVs as reported in Table 2 (Mirza et al. 1979; Mirza and MacGregor 1979; Melchers 1999; Simões da Silva et al. 2009; Ubejd Mujagi¢ and Easterling 2009).

The structure is discretized uniformly into eight 10-DOF elements with five Gauss-Lobatto points each. A quasi-static, monotonic, materially nonlinear-only analysis of the beam structure is performed using the Newton-Raphson incremental-iterative procedure in displacement-control mode with the vertical displacement at the point of application of the load taken as the controlled DOF, thus mimicking the physical experiment. The midspan vertical deflection of the beam is incremented from 0.0 to 65.0 mm, with increments of 0.5 mm. The 65.0 mm value of midspan deflection corresponds to the experimental failure of the physical specimen and is used as largest midspan deflection value for the FOSM analysis. In the nonlinear analysis, the applied force is obtained as the opposite of the internal resisting force exerted by the FE model at the controlled DOF. Thus, although the midspan deflection is a deterministic variable, the applied force P is a random variable and its mean value, μ_P, and standard deviation, σ_P, are computed and shown in the following figures. The analyses of both benchmark problems are performed using FEDEASLab (Filippou and Constantinides 2004) in the version extended for DDM-based FE response sensitivity analysis (Franchin 2004; Conte et al. 2004; Zona et al. 2005, 2006; Barbato et al. 2007). FEDEASLab is a Matlab (Mathworks 1997) toolbox suitable for linear and nonlinear, static and dynamic FE analysis of structural models.

Table 1. Additional Data for Beam 2 Tested by Ansourian (1982)

Test	Concrete compressive strength on 200-mm cube (MPa)	Lower yield stress for the flange (MPa)	Lower yield stress for the web (MPa)
1	20.000	367.7085	384.4156
2	19.375	361.0229	363.6364
3	20.938	363.9371	374.3316
4	—	372.6065	—
Mean	20.10433	366.3188	374.1278
COV (%)	3.9	1.4	2.8

Table 2. Mean and COV Values for the Material Parameters Used in the Benchmark Examples

Materials	Material parameter	Beam 2		Beam CTB1	
		Mean	COV (%)	Mean	COV (%)
Concrete	f_c (MPa)	20.1	3.9	24.6	20.0
	f_0 (MPa)	8.3	20.0	15.0	20.0
	E_c (MPa)	25,457	20.0	29,000	20.0
	ε_P (-)	0.0022	20.0	0.0020	20.0
	ε_0 (-)	0.0055	20.0	0.0045	20.0
Reinforcement steel	$f_{y0,\mathrm{reinf}}$ (MPa)	430.0	10.6	430.0	10.6
	$E_{0,\mathrm{reinf}}$ (MPa)	200,000	3.3	200,000	3.3
	b_{reinf} (-)	0.005	15.0	0.003	15.0
Girder steel	$f_{y0,\mathrm{girder}}$ (MPa)	—	—	277.0	10.0
	$f_{y0,\mathrm{flange}}$ (MPa)	366.3	1.4	—	—
	$f_{y0,\mathrm{web}}$ (MPa)	374.1	2.8	—	—
	$E_{0,\mathrm{girder}}$ (MPa)	200,000	3.0	200,000	3.0
	b_{girder} (-)	0.008	25.0	0.005	25.0
Connection	$f_{s,\max}$ [kN]	667	20.0	653	20.0

In Fig. 4, the experimental results and the mean response obtained by using FOSM analysis are compared with the mean response obtained through MCS based on 100 realizations of the set of all material parameters modeled as random variables. In the MCS, consistently with the statistical information available, three different modeling assumptions for the uncertainty of the material parameters are considered, i.e., material parameters represented by (1) a single lognormal random variable for the entire structure (LN-∞); (2) one lognormal random variable for each

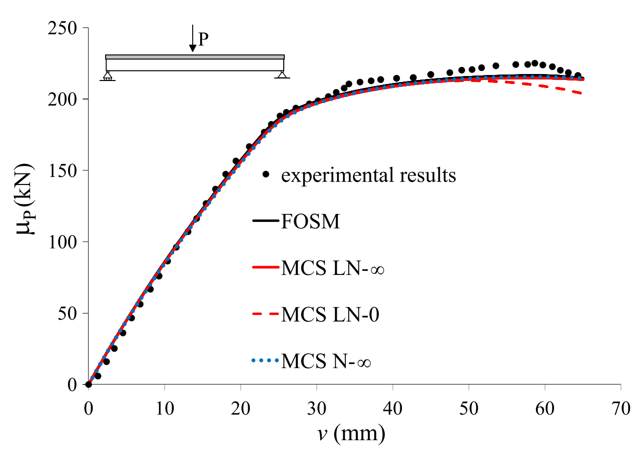

Fig. 4. Simply supported beam (Beam 2): comparison between experimental results and mean probabilistic force-displacement response obtained using different modeling hypotheses for the material parameters uncertainty

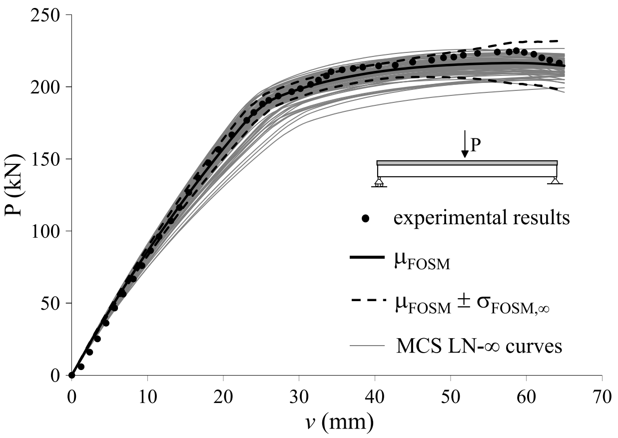

Fig. 5. Simply supported beam (Beam 2): comparison between experimental results and numerical probabilistic force-displacement response

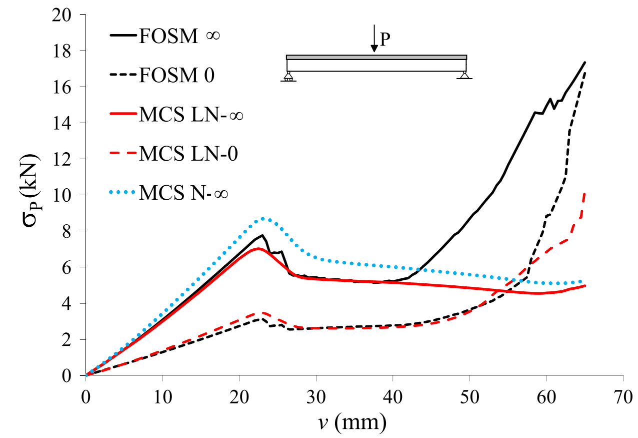

Fig. 6. Simply supported beam (Beam 2): comparison of the standard deviation estimates of the applied force obtained using different modeling hypotheses for the material parameters uncertainty

FE used to discretize the structure (LN-0); and (3) a single normal random variable for the entire structure (N-∞). The results obtained from MCS for the three modeling assumptions are very similar, except for a small deviation of the LN-0 case at large deflections, and they are close to both the experimental and FOSM results.

Fig. 5 shows the comparison of the experimental applied force-midspan deflection response (i.e., push-down curve) and the mean response (μ) as well as the mean response $\pm$ one standard deviation ($\mu \pm \sigma$) obtained numerically using the DDM-based FOSM method with no spatial variability for all the material parameters (assumption affecting the FOSM estimate of the response standard deviation as discussed previously). To illustrate the variability of the FE response simulation, the 100 realizations of the midspan force-deflection response curves obtained through MCS under the LN-∞ assumption are also plotted in Fig. 5. The predicted mean push-down curve is in very good agreement with the experimental results. Except at very small values of the vertical displacement, the experimental results are always contained between the $\mu - \sigma$ and $\mu + \sigma$ push-down curves. The response uncertainty is found to be small and of the same order of magnitude of the actual COVs (obtained from tests on samples of materials that the beam was made of) of the material parameters that most affect the structural response, namely f_c and f_{y0}.

Fig. 6 shows the numerical estimates of the standard deviation, σ_P, of the applied force P (which is a response parameter in the present displacement-controlled analysis) obtained by using (1) DDM-based FOSM analysis with no spatial variability (FOSM ∞); (2) DDM-based FOSM analysis with significant spatial variability (FOSM 0); (3) MCS under the LN-∞ assumption; (4) MCS under the LN-0 assumption; and (5) MCS under the N-∞ assumption. It is observed that the FOSM estimate of the response standard deviation σ_P is significantly affected by the spatial variability of the material parameters. The response standard deviation is significantly lower for the LN-0 case than for the LN-∞ and N-∞ cases for deflections up to approximatelyh 50 mm and becomes larger for midspan deflections higher than 56 mm. The differences between the LN-∞ and N-∞ cases are smaller, although non-negligible, with the response standard deviation, σ_P, larger for the N-∞ case than for the LN-∞ case. Notably, the standard deviation of the applied force P decreases noticeably for midspan deflections in the range between 23 and 28 mm. This phenomenon is because, at approximately 23 mm of midspan deflection, the simply supported beam starts experiencing yielding of both the shear connection (near the supports) and the steel beam (at midspan). After the shear connection yielding and the steel beam yielding spread along the SCC beam and across the steel beam section, respectively, the uncertainties in the connection shear strength and Young's modulus of the steel beam provide smaller contributions to the total response variability. This phenomenon results from a drop of the absolute value of the sensitivity of the SCC beam response to these two parameters in this range of response behavior of the beam. The response standard deviation obtained using FOSM analysis agrees well with the corresponding results obtained using MCS up to deflections of 42 mm for the LN-∞ and N-∞ assumptions, and 58 mm for the LN-0 assumption. This finding is consistent with other results reported in the literature (Barbato et al. 2010), showing that FOSM analysis is accurate for low-to-moderate level of material nonlinearity in the FE response.

For this specific example, the computational cost of the DDM-based FOSM method is approximately three times that of a deterministic FE analysis for the assumption of no spatial variability, and approximately 16.5 times the computational cost of a deterministic FE analysis for the assumption of significant spatial variability. Thus, the DDM-based FOSM method is significantly more efficient computationally than MCS, which requires 100 simulations for a COV of 1–2% on the estimated mean and 7–8% on the estimated standard deviation.

Nonsymmetric Two-Span Continuous Beam

The second benchmark problem considered is a nonsymmetric two-span continuous beam [Fig. 3(b)] tested by Ansourian (1981) under monotonic quasi-static loading and referred to as CTB1. This beam has two spans of length 4.00 and 5.00 m, respectively, and is subjected to a vertical concentrated load P applied at the midpoint of the shorter span. The joist section is a European IPE200, and the RC slab section is 100×800 mm^2. Shear lag effects are also neglected in this case. This benchmark example is particularly interesting because it reproduces the main phenomena typically observed in the ultimate strength analysis of SCC beams, such as concrete softening in compression, concrete cracking in tension, and high gradients of slip and shear force along the connection (Dall'Asta and Zona 2002). The reader is referred to Ansourian (1981) for all details regarding the geometry and material properties of this specimen. The mean and COV of each of the material parameters for this example are given in Table 2. In this case, the mean values of the material parameters were directly taken or derived from information published in Ansourian (1981). Because neither the COV of the material parameters nor the results from the individual material uniaxial and shear tests were published or recovered by Ansourian, typical values suggested or used in the literature were assumed for the COVs (Mirza et al. 1979; Mirza and MacGregor 1979; Melchers 1999; Simões da Silva et al. 2009; Ubejd Mujagi¢ and Easterling 2009).

The structure is discretized uniformly using 18 10-DOF elements with five Gauss-Lobatto points each. As in the previous application example, a quasi-static, monotonic, materially nonlinear-only analysis of the beam structure is performed using the Newton-Raphson incremental-iterative procedure in displacement-control mode with the vertical displacement at the point of application of the load taken as the controlled DOF; thus, closely mimicking the physical experiment. The vertical deflection of the beam at the midpoint of the shorter span (referred to as midspan deflection in the following) is incremented from 0.0 to 53.0 mm, with increments of 0.5 mm. The 53.0-mm value of midspan deflection corresponds to the experimental failure of the physical specimen and is used as largest midspan deflection value for the FOSM analysis. Thus, also in this second benchmark, the midspan deflection is a deterministic variable, whereas the applied force is a random variable obtained as the opposite of the resisting force exerted by the FE model at the controlled DOF.

Fig. 7 compares the experimental results and the mean response obtained using FOSM analysis with the mean response obtained through MCS based on 100 realizations of the set of all material parameters modeled as random variables. As in the previous application example, three different modeling assumptions for the uncertainty of the material parameters (i.e., LN-∞, LN-0 and N-∞) are considered in the MCS. These three uncertainty modeling assumptions are admissible based on the incomplete statistical information available. The mean response results obtained from FOSM analysis and N-∞ MCS practically coincide (with small differences for deflections larger than 35 mm), whereas those obtained using LN-∞ MCS and LN-0 MCS are slightly lower. Also in this example, all the numerically estimated mean responses are very close to the experimental results.

Fig. 8 shows the experimental applied force-midspan deflection response (push-down curve) and the mean response (μ) as well as the mean response $\pm$ one standard deviation ($\mu \pm \sigma$) obtained numerically using the DDM-based FOSM method assuming no spatial variability for all the material parameters. To illustrate the variability of the FE simulated response, the 100 MCS realizations

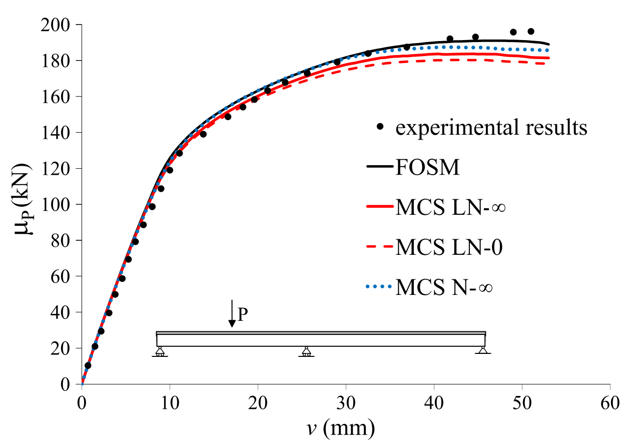

Fig. 7. Two-span continuous beam (Beam CTB1): comparison between experimental results and mean probabilistic force-displacement response obtained using different modeling hypotheses for the material parameters uncertainty

(under the LN-∞ assumption) of the midspan force-deflection response curves are also plotted in Fig. 8. The analytically predicted mean push-down curve is in excellent agreement with the experimental results. Also in this case, the experimental results are contained between the $\mu - \sigma$ and $\mu + \sigma$ push-down curves. These results, as well as the corresponding results for the simply supported beam considered in the previous benchmark example, suggest that the FE models adopted in this study, based on the mean values of the material parameters, are able to capture very well the actual load-deflection behavior of simply supported and continuous SCC beams. For this application example, experimental data are available only for the mean values of the material model parameters, whereas the standard deviations are obtained from typical values of the COV presented in the literature for these material parameters. These COVs are significantly larger than those obtained directly from experimental data in the previous benchmark example. Thus, the numerically estimated response COV is significantly larger than in the previous example.

Fig. 9 shows the numerical estimates of the standard deviation of the applied force obtained by using (1) DDM-based FOSM analysis under the assumption of no spatial variability; (2) DDM-based FOSM analysis under the assumption of significant spatial variability; (3) MCS analysis under the LN-∞ assumption;

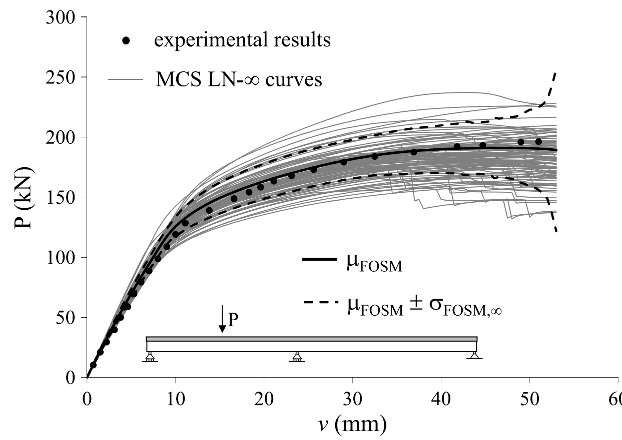

Fig. 8. Two-span continuous beam (Beam CTB1): comparison between experimental results and numerical probabilistic force-displacement response

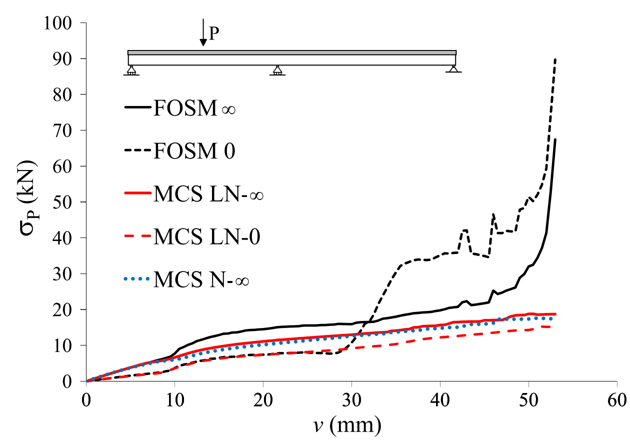

Fig. 9. Two-span continuous beam (Beam CTB1): comparison of the standard deviation estimates of the applied force obtained using different modeling hypotheses for the material parameters uncertainty

(4) MCS analysis under the LN-0 assumption; and (5) MCS analysis under the N-∞ assumption. The estimate of the response standard deviation obtained by using FOSM for the assumption of no spatial variability almost coincides with the MCS LN-∞ and N-∞ results for deflections up to approximately 10 mm, and it is 35% larger on average than the MCS LN-∞ and N-∞ results for deflections between 20 and 45 mm. For midspan deflections larger than 45 mm, at which the level of material nonlinearity in the response is very high, the FOSM approximation for the assumption of no spatial variability increases drastically with the midspan deflection compared to the MCS results, indicating a significant loss of accuracy of the FOSM approximation that is unable to capture the highly nonlinear dependency of the structural response on the material parameters. The differences between the LN-∞ and N-∞ cases are small but non-negligible. Unlike in the first application example, the MCS N-∞ standard deviation is smaller than the MCS LN-∞ standard deviation over the entire range of midspan deflections considered. The estimate of the response standard deviation obtained by using FOSM for the assumption of significant spatial variability almost coincides with the MCS LN-0 results for deflections up to approximately 29 mm, whereas this FOSM estimate increases drastically and departs from the MCS LN-0 results at larger midspan deflections, showing a significant loss of accuracy of the FOSM approximation even more pronounced than in the LN-∞ and N-∞ cases.

Fig. 10 shows the percent relative contributions (defined as $(\Delta\sigma_v^2)_{ij}/\sigma_v^2$, where $(\Delta\sigma_v^2)_{ij} = \rho_{ij} \cdot (\partial v/\partial\theta_i) \cdot \sigma_i \cdot (\partial v/\partial\theta_j) \cdot \sigma_j$, $i, j = 1, 2, \ldots, m$) of the different random parameters to the total variance of the applied force as a function of the midspan deflection, under the LN-∞ assumption (Barbato et al. 2010). These results show that: (1) for small midspan deflections (lower than 10 mm), the response variability is controlled almost exclusively by $f_{s,\max}$ (which determines the initial stiffness of the shear connection) (Ollgaard et al. 1971; Zona et al. 2006) and the stiffness-related parameters E_c and E_0; (2) after yielding of the steel beam component and up to approximately 50 mm of midspan deflection, parameter f_{y0} controls the response variability; and (3) for very large midspan deflections (larger than 50 mm), parameters ε_0 and f_0 (which are related to the concrete crushing behavior) together with $f_{s,\max}$ become dominant in controlling the response variability. Also, the contribution to the response variability of the statistical correlation of f_c and f_0 becomes also significant for very large

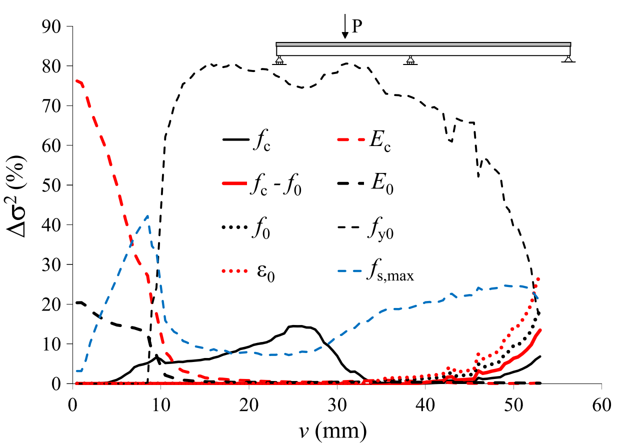

Fig. 10. Two-span continuous beam (Beam CTB1): Percentage relative contributions of the different individual random parameters and pairs of correlated random parameters to the variance of the applied force (LN-∞ assumption)

midspan deflections. The parameter relative contribution (for both individual and pairs of correlated random parameters) to the response variance is a valuable importance measure that is obtained at a negligible computational cost as by-product of a FOSM probabilistic response analysis (Barbato et al. 2010). Other similar importance measures (also obtained as a by-product of a FOSM probabilistic response analysis) are the tornado diagrams and the response sensitivities normalized in a deterministic and/or probabilistic sense (Barbato et al. 2010).

For this second example, the computational cost of the DDM-based FOSM method is approximately three times that of a deterministic FE analysis for the assumption of no spatial variability, and approximately 33 times the computational cost of a deterministic FE analysis for the assumption of significant spatial variability. Thus, the DDM-based FOSM method is computationally significantly more efficient than MCS, which requires 100 simulations for a COV lower than 2% on the estimated mean and lower than 8% on the estimated standard deviation. However, the computational cost advantage of using the DDM-based FOSM method decreases almost linearly with the number of random variables needed to discretize the random fields representing the material model parameters.

Current Challenges in Probabilistic Nonlinear Finite-Element Response Analysis

A crucial challenge faced with the combination of nonlinear FE analysis and probabilistic response analysis is the issue of numerical convergence of the FE response analysis, which can be hampered owing to material degradation (e.g., material softening and/or brittleness). This issue is particularly serious when MCS is used, because the simulation can produce combinations of parameter values for which the FE analysis encounters severe convergence difficulties. For example, in this study, among the one hundred MCS realizations used for each case, convergence difficulties are experienced (1) in nine analyses for the LN-∞ case, 24 analyses for the LN-0 case, and two analyses for the N-∞ case related to the Beam 2 benchmark; and (2) in 40 analyses for the LN-∞ case, 45 analyses for the LN-0 case, and 35 analyses for the N-∞ case related to the CTB1 Beam benchmark. These FE analyses required the use of an adaptive stepping technique (with increments of the controlled displacement reaching very small values in several cases, e.g., nominal increment divided by $2^{14} = 16{,}384$), which can increase several-fold the computational cost of a single FE analysis. This approach was computationally intensive for the two small-scale benchmark structures considered in this study, but would become unfeasible for large-scale real-world structural systems made up of hundreds or thousands of components. At the same time, discarding the MCS realizations that encounter convergence problems is not acceptable because it can significantly bias the FE response statistics. In general, among the subset of nonconverged realizations, distinguishing between lack of convergence resulting from numerical issues or because of physical failure of the structure analyzed can be a daunting if not impossible task. Another challenge faced by the FOSM method in capturing the statistics of the actual physical structural response is the spurious high sensitivity of the FE response and/or response sensitivities to numerical issues (e.g., numerical inversion of almost singular stiffness matrices, discontinuities in the FE response sensitivities owing to nonsmoothness of the response caused by material state transitions from elastic to plastic or brittle strength loss), which depend primarily on the numerical representation of the material behavior. Smoothing techniques applied to material constitutive models have been shown to effectively reduce the detrimental effects of some of the numerical issues mentioned previously (Barbato and Conte 2006).

Conclusions

This study employs a probabilistic response analysis methodology based on the use of the first-order second moment (FOSM) method in conjunction with the direct differentiation method (DDM) for response sensitivity computation to evaluate the variability of the structural response of steel-concrete composite (SCC) beams. This methodology is applied to compute the first-order and second-order statistical moments of the response of two actual structural systems (i.e., a simply supported SCC beam and a nonsymmetric two-span continuous SCC beam) for which experimental data on the nonlinear structural response are available. For the simply supported SCC beam, additional data are also available on the actual variability of concrete compressive strength and steel yield stress, which are two of the parameters to which the structural response is most sensitive. The DDM-based FOSM method results are compared with the available experimental measurements for the structural response and with the response statistics obtained using the computationally more expensive Monte Carlo-simulation (MCS) method, considering the effects of different modeling hypotheses for the material parameters uncertainty. For the examples considered in this paper, the DDM-based FOSM method agrees very well with the MCS method, for low-to-moderate levels of material nonlinearity in the response when the uncertainty of the material model parameters is moderate, and up to high levels of response nonlinearity when the material parameters uncertainty is small. The inaccuracy of the DDM-based FOSM method observed at high levels of material nonlinearity in the response is attributable to the increasingly nonlinear dependency of the response on the random variables describing the material parameters. Notably, for moderate-to-high response nonlinearity, MCS can be affected by computational issues because numerical convergence of the Monte Carlo realizations typically becomes more difficult and computationally demanding.

The DDM-based FOSM method is able to capture the effect of the spatial variability of the material parameters on the approximate structural response variance, but with a computational cost increasing with the total number of random variables describing the material parameters. Through MCS, it was found that, for the two examples considered in this paper, the spatial variability of the material parameters has a significant effect on the second-order statistical moment (variance) of the structural response, whereas its influence is negligible for the mean response. The FOSM method cannot account for the specific probability distributions of the material parameters. However, MCS shows that different material parameter distributions have very small effects on both the first-order and second-order statistical moments of the response of the structures considered in this study. The results presented in this paper, albeit obtained for a limited number of SCC beams, suggest that for this structural typology the spatial variability of the material parameters affects the variability of the structural response significantly more than the distribution type of the random variables describing the material parameters. Thus, when planning an experimental test to evaluate the response variability of a given SCC beam, it may be advisable to include material tests to evaluate the spatial variability of the material parameters instead of increasing the number of data points needed to estimate the distribution type of the random variables. Regarding the required computational time, the DDM-based FOSM method is significantly more efficient than MCS for the application examples considered in this paper. However, its cost increases linearly with the number of random variables used to describe the random material properties for the structural system of interest.

Acknowledgments

Partial supports of this research by (1) the National Science Foundation under Grant No. CMS-0010112; (2) the Pacific Earthquake Engineering Research (PEER) Center's Transportation Systems Research Program under Award No. 00006493; and (3) the Louisiana Board of Regents through the Pilot Funding for New Research (Pfund) Program of the National Science Foundation Experimental Program to Stimulate Competitive Research (EPSCoR) under Award No. NSF(2008)-PFUND-86, are gratefully acknowledged. The writers wish to thank Professor Peter Ansourian of The University of Sydney, for providing unpublished data from his experimental tests on simply supported beams. Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the writers and do not necessarily reflect the views of the sponsoring agencies.

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