Efficient Approach for the Reliability-Based Design of Linear Damping Devices for Seismic Protection of Buildings

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Abstract: This paper presents an efficient reliability-based methodology for the seismic design of viscous/viscoelastic dissipative devices in independent and/or coupled buildings. The proposed methodology is consistent with modern performance-based earthquake engineering frameworks and explicitly considers the uncertainties affecting the seismic input and the model parameters, as well as the correlation between multiple limit states. The proposed methodology casts the problem of the dampers' design for a target performance objective in the form of a reliability-based optimization problem with a probabilistic constraint. The general approach proposed in this study is specialized to stochastic seismic excitations and performance levels for which the structural behavior can be assumed as linear elastic. Under these conditions, the optimization problem is solved efficiently by taking advantage of existing analytical techniques for estimating the system reliability. This analytical design solution is an approximation of the optimal design and can be used as a hot-start point for simulation-based techniques, which can be employed to find the optimal design solution. An efficient correction formula is proposed to obtain an improved design solution that is generally sufficiently close for engineering purposes to the optimal design solution obtained from significantly more-computationally-expensive simulation-based techniques. The proposed design methodology is illustrated and validated by considering two steel buildings modeled as linear elastic multiple-degree-of-freedom systems for different linear damper properties and collocation, for both independent and coupled configurations. **DOI: 10.1061/AJRUA6.0000858.** © *2015 American Society of Civil Engineers.*

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Introduction

The use of supplemental damping in the form of viscous or viscoelastic dampers has become increasingly widespread in the design and retrofit of civil structures excited by earthquake loads because it often permits mitigation of undesirable aspects of the structural response at a lower cost than other more-traditional approaches. Experimental and analytical studies have demonstrated that the addition of viscous or viscoelastic dampers inside a building (Soong and Spencer 2002; Takewaki 2009) and/or between adjacent buildings (Zhang and Xu 1999; Kim et al. 2006; Roh et al. 2011; Tubaldi 2015) permits control of motion amplitude, interstory drifts, and absolute accelerations induced by earthquake actions.

In recent years, different methodologies for the optimal design of these damping devices under uncertain seismic input were proposed. However, significant simplifications were often introduced to reduce the complexity of the problem. Several of the design methods proposed in the literature (Shukla and Datta 1999; Takewaki 2009; Zhu et al. 2011; Richardson et al. 2013) expressed the performance objectives in terms of mean-square displacement or interstory drift response of the buildings, without explicit reliability considerations in terms of damage and loss risk. Numerous studies employed simple stochastic models (e.g., stationary white noise or Kanai-Tajimi models) to describe the seismic input, by disregarding its nonstationary characteristics (e.g., Bhaskararao and Jangid 2007; Takewaki 2009; Taflanidis and Scruggs 2010; Taflanidis 2010; Zhu et al. 2011). Only a few studies considered explicitly the effects of (1) model parameter uncertainty (MPU) (i.e., the uncertainty affecting the parameters used to define both the structural model and/or the limit states), which can have a nonnegligible influence on the structural performance (Guo et al. 2002; Taflanidis 2010); and (2) correlation between different component response and failure modes in evaluating the system reliability (Taflanidis 2010; Taflanidis and Scruggs 2010). The latter effects may significantly influence the seismic reliability estimates for a given system, especially in the case of adjacent buildings connected by dampers (Tubaldi et al. 2014).

In the last decade, significant progress was made towards overcoming the aforementioned limitations in reliability-based design procedures for passively-damped buildings. Marano et al. (2007) developed a reliability-based design approach for linear multistory frames protected by using linear viscous dampers. This approach was based on the minimization of a deterministic objective function defined as the total added damping, while stochastic constraints were imposed to limit the system failure probability for a given earthquake hazard level. Taflanidis (2010) presented a reliability-based methodology for the optimal design of systems subjected to

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stationary stochastic loading with uncertain model parameters. The study highlighted the importance of the correlation between the failure modes and MPU from a design perspective. Jensen and Sepulveda (2012) proposed a method for the design of structures equipped with passive dissipation systems by considering both record-to-record variability and MPU. The damper design was formulated as an optimization problem with a single objective function and multiple reliability constraints. The problem solution was sought through a sequential optimization approach (Jensen and Sepulveda 2011) that involved a converging series of approximate reliability analyses.

The present study presents a simple yet comprehensive probabilistic methodology for the reliability-based design of viscous/viscoelastic dampers that are added to structural systems subject to seismic hazard. This methodology explicitly considers the uncertainties affecting the seismic input and the model parameters, as well as the correlation between multiple limit states. The problem of the design for a target performance objective is cast in the form of a reliability-based optimization problem. Although the proposed methodology is general, in this paper it is specialized to the case of seismic excitation modeled as nonstationary stochastic processes, and performance objectives for which the structural behavior can be assumed linear elastic. This specialization permits taking advantage of recently derived time-variant reliability analysis techniques (Barbato and Conte 2008; Barbato and Vasta 2010; Barbato and Conte 2011, 2014) for evaluating the system risk of deterministic systems, and obtaining an approximate optimal design at a computational cost that is several orders of magnitude lower than that required to obtain the optimal design by using simulation-based techniques. A correction formula is also proposed to derive an improved design solution at a very low computational cost. The proposed design methodology is illustrated and validated by considering two steel buildings modeled as linear elastic multiple-degree-of-freedom systems under different configurations and design conditions.

Problem Formulation

Failure Probability Estimate for Systems Equipped with Damping Devices

The basis of the proposed design method is the efficient solution of the direct reliability problem, which corresponds to evaluating the failure probability, P_{f,t_L}, of the system with added dampers during its design life, t_L, i.e., the probability of not satisfying the performance objective during t_L. In this type of problem, the building's and dampers' properties, the seismic input characteristics, and the seismic hazard at the site are known quantities.

Under the assumption that the failure events can be described as a Poisson process and that the buildings are immediately restored to their original condition after failure, P_{f,t_L} is obtained as

$$P_{f,t_L} = 1 - e^{-\nu_f \cdot t_L} \quad (1)$$

in which ν_f = mean annual frequency (MAF) of system failure, which can be computed through the following convolution integral:

$$\nu_f = \int_{im} P_{f|IM}(im) \cdot |d\nu_{IM}(im)| \quad (2)$$

where $P_{f|IM}(im)$ = fragility function expressing the probability of system failure conditional to the seismic intensity measure $IM = im$; and $\nu_{IM}(im)$ = MAF of exceedance of a specific value im of IM. For the sake of simplicity, this study considers only scalar seismic intensity measures. The most challenging task of this direct reliability problem is the computation of the conditional failure probability, $P_{f|IM}(im)$. The reliability of multicomponent structural systems, such as those considered in this study, depends on the reliability of its components, i.e., the building's structural elements and the dampers. The computation of the conditional failure probability of the components, $P_{f_i|IM}(im)$ ($i = 1,2,\ldots,N_{ls}$, where N_{ls} = number of component limit states), and of the system, $P_{f|IM}(im)$, requires solving a time-variant reliability problem by accounting for all pertinent sources of uncertainty. The simplest approach for solving this time-variant reliability problem is Monte Carlo simulation (MCS). However, this approach can be computationally expensive, since it requires a very (sometimes prohibitively) large number of time-history analyses to obtain accurate results when small failure probabilities need to be estimated. Thus, advanced simulation techniques (Au and Beck 2001a, b) or analytical techniques based on random vibration theory (Guo et al. 2002; Park et al. 2004; Marano et al. 2007; Taflanidis 2010; Tubaldi et al. 2014) are usually preferred to MCS for practical structural engineering applications.

Reliability-Based Design of Damping Devices

The design of the damping devices that are needed to achieve a target system failure probability over its design life can be cast as an optimization problem, which identifies the optimal dampers properties (e.g., the properties that minimize a specified objective function) that also satisfy the stochastic constraints on the probability of exceeding a prescribed damage level. Additional constraints are needed to ensure that the dampers' properties assume physically-admissible values. The objective function depends on the type of device considered. For example, if a viscoelastic material such as rubber is employed, the dampers' cost can be assumed proportional to the rubber volume, which can be expressed as a function of the dampers' stiffness and geometric parameters (Park et al. 2004); whereas, for linear viscous dampers (e.g., fluid dampers), the sum of the dampers' viscous constants can be employed as a simplified approximation for the dampers' cost (Takewaki 2009). A more-accurate cost estimate could also consider the dampers' peak force and stroke (Hwang et al. 2013). However, the introduction of these variables would require a stochastic objective function, whose treatment is considered out of the scope of this paper. In this study, the design problem is mathematically formalized as follows:

$$\min_{\mathbf{d}} C(\mathbf{d}) \qquad \text{subject to } \mathbf{f}(\mathbf{d}) \leq 0 \qquad P_{f,t_L}(\mathbf{d}) - \bar{P}_f \leq 0 \quad (3)$$

where $\mathbf{d} = [k_{d,1}, \ldots, k_{d,m}, c_{d,1}, \ldots, c_{d,m}, a_{d,1}, \ldots, a_{d,m}]^T$ = vector of design variables; $k_{d,i}$, $c_{d,i}$, and $a_{d,i}$ ($i = 1,2,\ldots,m$) = stiffness, damping constant, and parameter describing the geometry, respectively, of the ith damper; the superscript T = matrix/vector transposition; m = total number of dampers; $\mathbf{f}(\mathbf{d}) \leq 0$ = additional (linear and/or nonlinear) deterministic constraints specifying the feasible domain of the damper properties; $C(\mathbf{d})$ = deterministic objective function; and $\bar{P}_f$ = target (design) failure probability.

The evaluation of $P_{f,t_L}(\mathbf{d})$ (in which the explicit dependency on the design variables $\mathbf{d}$ is shown for clarity) is the most computationally-demanding task in the design procedure corresponding to Eq. (3). This study focuses on the development of efficient solution techniques based on the assumption of linear elastic structural behavior. In fact, the dampers are often employed to achieve performance levels corresponding to negligible structural damage, e.g., immediate occupancy or operational performance level, as defined in FEMA 273 (FEMA 1997) and

FEMA 356 (FEMA 2000). For these performance levels, the assumption of linear elastic structural behavior is satisfied.

Efficient Solution of the Reliability-Based Design of Structural Systems with Added Dampers

This section describes an efficient methodology for solving the reliability-based design problem defined by Eqs. (1)–(3) for performance levels corresponding to linear elastic behavior of the structural systems under consideration. First, the equations of motion of a general system of two building coupled using viscous/viscoelastic dampers and the seismic input model are introduced. Then, an efficient methodology for obtaining an approximate solution for the design problem expressed by Eq. (3) is presented. Finally, a correction formula to improve the approximate optimal design at a very low computational cost is proposed.

Equations of Motion for Coupled Buildings with Added Dampers

Under the assumption of linear elastic behavior, the equations of motion of two adjacent buildings with dampers added both inside and between the building's frames (Fig. 1) can be written as follows:

$$\mathbf{M}\cdot\ddot{\mathbf{U}}(t)+(\mathbf{C}+\mathbf{C}_d)\cdot\dot{\mathbf{U}}(t)+(\mathbf{K}+\mathbf{K}_d)\cdot\mathbf{U}(t)=-\mathbf{M}\cdot\mathbf{R}\cdot\ddot{\mathbf{U}}_g(t) \tag{4}$$

in which $\mathbf{U}=\begin{bmatrix}\mathbf{U}_\mathrm{A}\\ \mathbf{U}_\mathrm{B}\end{bmatrix}$; $\mathbf{M}=\begin{bmatrix}\mathbf{M}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_\mathrm{B}\end{bmatrix}$; $\mathbf{K}=\begin{bmatrix}\mathbf{K}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{K}_\mathrm{B}\end{bmatrix}$; $\mathbf{C}=\begin{bmatrix}\mathbf{C}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_\mathrm{B}\end{bmatrix}$; $\mathbf{U}_i$ = displacement vector of the free degrees-of-freedom of building i $(i=A,B)$; $\mathbf{M}_i$, $\mathbf{K}_i$, and $\mathbf{C}_i$ = mass, stiffness, and damping matrices of building i $(i=A,B)$, respectively; $\mathbf{K}_d$ and $\mathbf{C}_d$ = stiffness and damping matrices corresponding to the added dampers, respectively; $\mathbf{R}$ = influence coefficient matrix; $\ddot{\mathbf{U}}_g(t)$ = vector containing the different components of the input ground motion; t = time; and a superposed dot denotes differentiation with respect to time. The equations of motion of a single building with added dampers are a particular subcase of Eq. (4).

Matrices $\mathbf{K}_d$ and $\mathbf{C}_d$ contain the information regarding both the properties and the location of the dampers within and/or between the buildings (Fig. 1). Any set of structural responses that can be obtained from the displacement response vector $\mathbf{U}(t)$ by means of a linear operator (e.g., interstory drifts, base shear, and floor shears) can be used as engineering demand parameter to monitor the response of the system components.

In this paper, the input ground-acceleration components are modeled as separable nonstationary stochastic processes (Barbato and Vasta 2010). This analytical representation of the seismic input is completely defined by a power spectral density (PSD) function of an embedded Gaussian stationary process and by a deterministic time-modulating function. The parameters needed to describe both the PSD and the time-modulating functions must be appropriately chosen in order to accurately represent the characteristics of the seismic input expected at the site (e.g., frequency content, spectral acceleration at specified frequencies, and input duration). The description of the seismic input is completed by an appropriate hazard function for the site, i.e., $\nu_{IM}(im)$. The IM should be selected based on sufficiency and efficiency criteria (Luco and Cornell 2007) and should be easily related to the stochastic description of the input ground motion process. If multiple active seismic sources can affect the site of interest, a different seismic ground-motion model can be used for each source.

Efficient Approximate Solution of the Reliability-Based Design Problem

A previous study (Tubaldi et al. 2014) proposed an analytical technique for the seismic risk assessment of adjacent buildings connected by linear and nonlinear viscous/viscoelastic dampers. In particular, the seismic risk P_{f,t_L} for deterministic systems can be efficiently and accurately approximated by (1) evaluating the component fragilities $P_{f_i|IM}(im)$ $(i=1,2,\ldots,N_{ls}$, where N_{ls} = number of component limit states) by solving a first-passage reliability problem through the use of approximate analytical time-variant hazard functions (i.e., based on the Poisson's, classical Vanmarcke's, or modified Vanmarcke's approximations); (2) estimating the system failure probability conditional to im, $P_{f|IM}(im)$, by using a series system idealization and assuming perfectly correlated limit states; and (3) computing the MAF of the system failure, ν_f, and the failure probability over the design life of the structural system, P_{f,t_L}, via Eqs. (2) and (1), respectively. In the case of systems with uncertain properties, the computation of $P_{f_i|IM}(im)$ $(i=1,2,\ldots,N_{ls})$ is performed by using the total probability theorem in conjunction with simulation techniques such as Latin hypercube sampling (LHS), as described in Tubaldi et al. (2014).

The optimization problem defined by Eq. (3) can have multiple local minima and, thus, must be solved using global optimization techniques. In this study, a multiple start-point algorithm based on gradient-based iterative local optimizers was employed (Ugray et al. 2007; *MATLAB*). Local optimization algorithms require computing repeatedly the system failure probability, $P_{f,t_L}(\mathbf{d})$, and its gradient with respect to the design variables $\mathbf{d}$, $\nabla_\mathbf{d}P_{f,t_L}(\mathbf{d})$. The analytical technique proposed in Tubaldi et al. (2014), referred to as analytical (AN) algorithm, provides a smooth representation of these quantities, denoted respectively as $P^{\mathrm{AN}}_{f,t_L}(\mathbf{d})$ and $\nabla_\mathbf{d}P^{\mathrm{AN}}_{f,t_L}(\mathbf{d})$, and can be efficiently used to calculate them. The AN algorithm neglects the effects of MPU and provides a fully analytical and computationally-efficient approximation of the reliability-based design problem. The assumption of deterministic system and the use of analytical estimates of the first-passage failure probability in the AN algorithm also permits to avoid numerical issues which may arise when stochastic simulation is used in conjunction with gradient-based optimization algorithms (Taflanidis and Beck 2008; Jensen et al. 2009). The local optimization algorithm is assumed to converge at iteration $i+1$ to the solution $\mathbf{d}^*_{\mathrm{AN},j}$ $(j=1,2,\ldots,n_{\mathrm{sp}}$, where n_{sp} = number of start points) when $[|C(\mathbf{d}_{i+1})-C(\mathbf{d}_i)|]/C(\mathbf{d}_i)<\varepsilon_1$ and $|P^{\mathrm{AN}}_{f,t_L}(\mathbf{d}_{i+1})-\bar{P}_f|<\varepsilon_2$

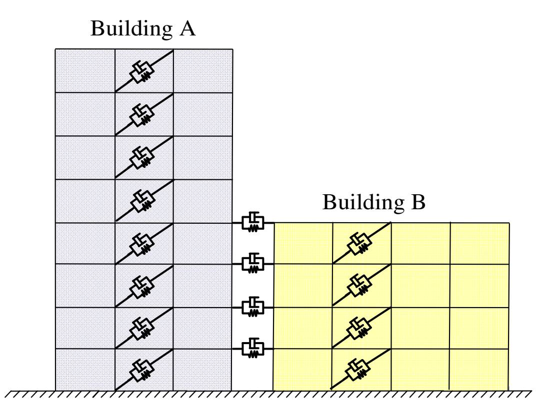

Fig. 1. Buildings equipped with viscoelastic dampers placed inside and between the buildings

respectively, where ε_1, ε_2 = user-defined tolerances. The design solution is given by $\mathbf{d}^*_{\mathrm{AN}} = \min_j(\mathbf{d}^*_{\mathrm{AN},j})$.

In general, due to the approximate nature of the time-variant hazard function, the assumptions made on the correlation among the failure modes, and the effects of MPU that are neglected by the AN algorithm, the stochastic constraint on the system failure probability in Eq. (3) may not be strictly satisfied by the approximate design solution $\mathbf{d}^*_{\mathrm{AN}}$, i.e., the condition $|P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}}) - \bar{P}_f| > \varepsilon_2$ may occur, where $P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})$ = system failure probability evaluated through MCS at $\mathbf{d}^*_{\mathrm{AN}}$. However, $\mathbf{d}^*_{\mathrm{AN}}$ can be used as a hot-start point for optimization algorithms based on stochastic simulation techniques, such as the simulation (SIM) algorithm and the hybrid (HYB) algorithms proposed in Barbato and Tubaldi (2013), whose solutions ($\mathbf{d}^*_{\mathrm{SIM}}$ and $\mathbf{d}^*_{\mathrm{HYB}}$, respectively) strictly satisfy the constraint on the system failure probability within a user-defined tolerance. The SIM algorithm is an iterative optimization algorithm that uses the estimate of the failure probability and its gradient obtained through stochastic simulation, respectively denoted to as $P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d})$ and $\nabla_{\mathbf{d}} P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d})$, whereas the HYB algorithm uses the estimate of the failure probability obtained through simulation, $P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d})$, in conjunction with the analytical estimate of the failure probability gradient, $\nabla_{\mathbf{d}} P^{\mathrm{AN}}_{f,t_L}(\mathbf{d})$. The computational cost of the SIM and HYB algorithms is several orders of magnitude higher than the computational cost of the AN algorithm. Furthermore, numerical problems often arise when stochastic simulation is used in conjunction with gradient-based algorithms (Taflanidis and Beck 2008; Jensen et al. 2009), which may further increase the number of iterations required to achieve convergence.

Design Correction Formula

In order to improve the approximate design solution obtained using the AN algorithm while avoiding the higher computational cost of the SIM and HYB algorithms, the following correction formula for the solution $\mathbf{d}^*_{\mathrm{AN}}$ is proposed in this study:

$$\mathbf{d}^*_{\mathrm{corr}} = \left[1 + \frac{\bar{P}_f - P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})}{\nabla_{\mathbf{d}} P^{\mathrm{AN}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}}) \cdot \mathbf{d}^*_{\mathrm{AN}}}\right] \cdot \mathbf{d}^*_{\mathrm{AN}} \tag{5}$$

The corrected approximate design point $\mathbf{d}^*_{\mathrm{corr}}$ is obtained by equating to $\bar{P}_f$ the first-order Taylor's series approximation of the failure probability about $\mathbf{d}^*_{\mathrm{AN}}$ along the direction defined by $\mathbf{d}^*_{\mathrm{AN}}$, with the gradient $\nabla_{\mathbf{d}} P^{\mathrm{AN}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})$ computed using the analytical approximation of the hazard function. Thus, the proposed correction formula corresponds to scaling the optimal damper properties found using the AN algorithm. Fig. 2 provides a graphical representation of the correction for a case involving two design variables. The proposed correction formula is based on two main assumptions: (1) the design solution obtained through the AN algorithm yields a damper distribution along the building height that is proportional to the distribution corresponding to the optimal design solution; and (2) the optimal design solution lies at the boundary of the reliability constraint, i.e., increasing the target failure probability always results in a decrease of the dampers cost. The results of the application examples presented in this study confirm that the second assumption is generally satisfied.

The computational cost of the proposed correction is mainly due to the computation of $P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})$ through stochastic simulation, whereas the computational cost of evaluating $\nabla_{\mathbf{d}} P^{\mathrm{AN}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})$ is almost negligible. The effects of the MPU on the optimal damper properties can also be included through Eq. (5) by using the LHS technique to evaluate $P^{\mathrm{SIM}}_{f,t_L}(\mathbf{d}^*_{\mathrm{AN}})$. In general, the proposed first-order correction formula provides a start point $\mathbf{d}^*_{\mathrm{corr}}$ for the SIM and

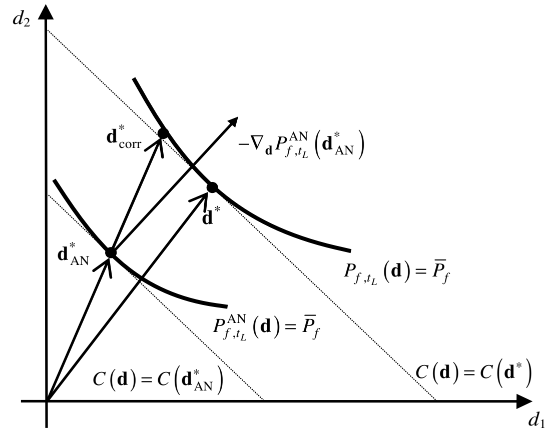

Fig. 2. Graphical representation of the correction formula

HYB algorithms that is closer to satisfying the stochastic constraint on the system failure probability when compared to $\mathbf{d}^*_{\mathrm{AN}}$, and is often sufficiently accurate for design purposes.

Application Examples

The reliability-based design methodology and correction formula developed in this study were applied to determine the optimal properties and location of viscous dampers under two different design scenarios: (1) viscous dampers located inside two adjacent buildings, and (2) viscous dampers connecting two adjacent buildings. In the second scenario, the effects of MPU are also considered.

The two adjacent buildings considered in this study were steel moment-resisting frames modeled as linear elastic multiple-degree-of-freedom shear-type systems (Fig. 1). The properties of these buildings, initially assumed as deterministic, were taken from Tubaldi et al. (2014). Building A was an eight-story frame with constant floor mass, $m_A = 454{,}540$ kg, and stiffness, $k_A = 628{,}801$ kN/m (Lin 2005). Building B was a four-story building with constant floor mass, $m_B = 454{,}540$ kg, and stiffness, $k_B = 470{,}840$ kN/m. The story heights were equal to $H = 3.2$ m. Matrices $\mathbf{C}_{\mathrm{A}}$ (with dimensions 8×8) and $\mathbf{C}_{\mathrm{B}}$ (with dimensions 4×4) describe the inherent buildings' damping. They were based on the Rayleigh model and were obtained by assuming a damping factor $\xi_A = \xi_B = 2\%$ for the first two vibration modes of each system. The fundamental vibration periods of Buildings A and B were $T_A = 0.915$ s and $T_B = 0.562$ s, respectively.

The stochastic seismic input considered in this study was modeled as an embedded stationary Gaussian process modulated in time through a Shinozuka-Sato's modulating function (Shinozuka and Sato 1967), i.e.

$$I(t) = c \cdot (e^{-b_1 \cdot t} - e^{-b_2 \cdot t}) \cdot H(t) \tag{6}$$

in which $b_1 = 0.045\pi\,\mathrm{s}^{-1}$; $b_2 = 0.050\pi\,\mathrm{s}^{-1}$; $c = 25.812$; and $H(t)$ unit step function. A duration $t_{\max} = 30$ s was assumed for the seismic excitation. The PSD function of the embedded stationary process was described by the widely-used Kanai-Tajimi model, as modified by Clough and Penzien (1993), i.e.

$$S_{CP}(\omega) = S_0 \cdot \frac{\omega_g^4 + 4 \cdot \xi_g^2 \cdot \omega^2 \cdot \omega_g^2}{[\omega_g^2 - \omega^2]^2 + 4 \cdot \xi_g^2 \cdot \omega^2 \cdot \omega_g^2} \cdot \frac{\omega^4}{[\omega_f^2 - \omega^2]^2 + 4 \cdot \xi_f^2 \cdot \omega^2 \cdot \omega_f^2} \tag{7}$$

in which S_0 = amplitude of the bedrock excitation spectrum, modeled as a white noise process; ω_g and ξ_g = fundamental circular frequency and damping factor of the soil, respectively; and ω_f and ξ_f = parameters describing the Clough-Penzien filter. The following values of the parameters were used: $\omega_g = 12.5$ rad/s, $\xi_g = 0.6$, $\omega_f = 2$ rad/s, and $\xi_f = 0.7$.

The peak ground acceleration, PGA, was assumed as IM. In order to derive the fragility curves in terms of the selected IM, the relationship between the parameter S_0 of the modified Kanai-Tajimi spectrum and the PGA at the site was assessed empirically based on the procedure shown in Tubaldi et al. (2012). The site seismic hazard curve was

$$\nu_{\text{PGA}}(\text{pga}) = 6.734 \cdot 10^{-5} \cdot \text{pga}^{-2.857} \tag{8}$$

This seismic hazard curve was chosen so that, for the site of interest, a PGA $= 0.3$ g (where g = gravity constant) corresponded to a probability of being exceeded equal to 10% in 50 years (i.e., to a return period of 475 years).

Design of Viscous Dampers Located inside the Buildings

The first application example consists of the design of linear viscous dampers inserted inside the building frames. The target performance objective was defined so that the probability P_{f,t_L} of exceeding the immediate occupancy performance level for the coupled system in $t_L = 50$ years must be less than or equal to $\bar{P}_f = 10\%$. The component limit states considered corresponded to the exceedance of the interstory drift limits of 0.7% by any of the two buildings' stories, as specified in FEMA 273 (FEMA 1997) and FEMA 356 (FEMA 2000). This interstory drift value can be considered as a conventional limit for the linear elastic behavior of multistory steel buildings and for their immediate occupancy limit state, which corresponds to negligible structural damage. The assumption of linear elastic behavior was deemed accurate for this specific performance level, since the fact that the buildings may experience nonlinear behavior for high (and rare) IM values was not expected to bias the risk estimates. The damper limit states were not considered in this application, since the dampers are commonly sized so that their failure probability is significantly smaller than the probability of exceedance of the 0.7% drift limit in the frame. In modern design codes, the performance objectives are usually expressed as required performance for a seismic event with a prescribed return period (FEMA 2000), e.g., as a maximum acceptable drift limit for an earthquake intensity with a given probability of being exceeded in 50 years. However, a rigorous reliability-based design approach requires selecting the values of the target failure probability for the maximum acceptable drift limits at each performance level of interest. Failure probability values and their selection commonly depend on the interacting needs of the different stakeholders; thus, the value of 10% failure probability in 50 years was selected in this study based on engineering judgment and economic considerations.

Without dampers, the probabilities of exceedance for the immediate occupancy performance level in 50 years, P_{f,t_L}, were 26.07 and 28.55%, for Buildings A and B, respectively. Two design options were investigated: (1) viscous dampers with uniform properties located at all building stories (referred to as uniform-distribution case); and (2) viscous dampers with properties variable from story to story (referred to as variable-distribution case). This latter damper distribution corresponded to eight design variables, i.e., $\mathbf{d} = [c_{d1}, \ldots, c_{d8}]^T$, for the 8-story building (Building A), and to four design variables, i.e., $\mathbf{d} = [c_{d1}, c_{d2}, c_{d3}, c_{d4}]^T$, for the 4-story building (Building B). The feasible domain for the damper viscous constants was described by the nonlinear constraints $c_{d,i} \cdot (c_{d,\min} - c_{d,i}) \leq 0$, where $c_{d,\min} = 200$ kN $\cdot$ s/m, and by the lower bound $c_{d,i} \geq 0$. These constraints ensured that, at any given floor, the optimal solution corresponds either to the case of no dampers or to values of $c_{d,i} \geq c_{d,\min}$, which was assumed as the minimum value of the damper viscous constant for which dampers were readily available at a market competitive cost.

The AN algorithm employed the modified Vanmarcke's approximation of the time-variant hazard function in conjunction with the assumption of perfect correlation between the failure modes to estimate $P_{f,t_L}^{\text{AN}}(\mathbf{d})$ and $\nabla_{\mathbf{d}} P_{f,t_L}^{\text{AN}}(\mathbf{d})$ during the iterations. The estimates of $P_{f,t_L}^{\text{SIM}}(\mathbf{d})$ required at each iteration of the SIM algorithm were obtained via MCS by considering 10,000 artificial ground-motion records compatible with the input PSD and generated through the spectral representation method (Shinozuka and Deodatis 1991). This number of samples ensured accurate estimates of the MAF of system failure, v_f, with a coefficient of variation of the estimate of v_f smaller than 1%. The tolerances for the design problem were selected as $\varepsilon_1 = \varepsilon_2 = 10^{-3}$. Multiple start points were considered for the AN algorithm to find the global minimum of the objective function. To limit the number of iterations needed for convergence, the AN algorithm's optimal solution, $\mathbf{d}^*_{\text{AN}}$, was used as a hot-start point for the SIM algorithm, the results of which were considered as the reference solution for this application example. In applying both AN and SIM algorithms, the failure probability's gradients [i.e., $\nabla_{\mathbf{d}} P_{f,t_L}^{\text{AN}}(\mathbf{d})$ and $\nabla_{\mathbf{d}} P_{f,t_L}^{\text{SIM}}(\mathbf{d})$, respectively] were computed at each iteration using the finite-difference method, i.e., by perturbing each component of vector $\mathbf{d}$ one at a time and calculating the corresponding change in failure probability. For this specific application example, the use of the modified Vanmarcke's approximation in conjunction with the assumption of perfect correlation between failure modes were shown to provide more-accurate estimates of the failure probability than other analytical approximations (Tubaldi et al. 2014). These approximations can affect the value of the optimal solution obtained through the AN algorithm, $\mathbf{d}^*_{\text{AN}}$, but have only a negligible effect on the value of reference optimal solution.

Tables 1 and 2 report the optimal design results obtained using the AN algorithm, the correction corresponding to Eq. (5), and the SIM algorithm for Buildings A and B, respectively. In all cases considered here, the design solution obtained using the AN algorithm is already close to the design solution obtained using the SIM algorithm (with a 7.8% difference in the uniform-distribution case and a 5.8% difference in the variable-distribution case for Building A, and a −8.3% difference in the uniform-distribution case and a −7.1% difference in the variable-distribution case for Building B). The proposed correction formula provides dampers' properties that are very close to those obtained through the SIM algorithm (with a 2.8% difference in the uniform-distribution case and a 0.9% difference in the variable-distribution case for Building A, and a 0.6% difference in the uniform-distribution case and a 0.4% difference in the variable-distribution case for Building B). It is observed that the design solution for the variable-distribution case requires dampers located only at the two lower stories, and allows reducing the cost associated with the retrofit (measured in terms of total added viscous damping) when compared to the design solution corresponding to the uniform-distribution case.

Figs. 3(a and b) report the component fragility curves (i.e., failure probabilities conditional to PGA) for the interstory drift ratios (IDRs) and the corresponding failure probabilities during the design life, respectively, for Building A. These estimates correspond to the solution obtained using the SIM algorithm by considering both the cases of uniform and variable distribution. The curves of

Table 1. Optimal Design of Viscous Dampers Placed inside Building A

Story number	Uniform distribution $\mathbf{d}^*_{AN}$ (kN · s/m)	Uniform distribution $\mathbf{d}^*_{corr}$ (kN · s/m)	Uniform distribution $\mathbf{d}^*_{SIM}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{AN}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{corr}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{SIM}$ (kN · s/m)
1	5,826.70	5,561.90	5,407.48	18,376.80	17,538.45	17,718.94
2	5,826.70	5,561.90	5,407.48	5,726.50	5,465.26	5,068.33
3	5,826.70	5,561.90	5,407.48	0	0	0
4	5,826.70	5,561.90	5,407.48	0	0	0
5	5,826.70	5,561.90	5,407.48	0	0	0
6	5,826.70	5,561.90	5,407.48	0	0	0
7	5,826.70	5,561.90	5,407.48	0	0	0
8	5,826.70	5,561.90	5,407.48	0	0	0
Sum	46,613.60	44,495.20	43,259.84	24,103.30	23,003.71	22,787.28
P_{f,t_L} (%)	9.65	9.92	10.05	9.71	9.95	10.00

Table 2. Optimal Design of Viscous Dampers Placed inside Building B

Story number	Uniform distribution $\mathbf{d}^*_{AN}$ (kN · s/m)	Uniform distribution $\mathbf{d}^*_{corr}$ (kN · s/m)	Uniform distribution $\mathbf{d}^*_{SIM}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{AN}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{corr}$ (kN · s/m)	Variable distribution $\mathbf{d}^*_{SIM}$ (kN · s/m)
1	2,631.50	2,886.91	2,869.38	6,041.30	6,524.60	6,500.90
2	2,631.50	2,886.91	2,869.38	0	0	0
3	2,631.50	2,886.91	2,869.38	0	0	0
4	2,631.50	2,886.91	2,869.38	0	0	0
Sum	10,526.00	11,547.65	11,477.52	6,041.30	6,524.60	6,500.90
P_{f,t_L} (%)	10.71	9.98	10.07	10.81	10.10	9.95

the system fragilities and of the system risk are very close to the corresponding curves for the first story and, thus, they are not reported in Figs. 3(a and b). This result is due to the fact that (1) the IDR demand at the first story is higher when compared to those at the other stories; and (2) the correlation between the IDR responses within the frame is very high. Figs. 4(a and b) plot the probabilities of exceedance in 50 years for the dampers' forces at the different stories of Building A, corresponding to the uniform-distribution and variable-distribution cases, respectively. It is observed that, for a given probability of exceedance, the sum of the dampers' forces in the variable-distribution case is significantly lower than the sum of the dampers' forces in the uniform-distribution case. Thus, the optimal design for the variable-distribution case is more efficient than the optimal design for the uniform-distribution case also in terms of the total forces acting in the dampers. Similar results to those presented in Figs. 3 and 4 were obtained also for Building B, but are not reported here due to space constraints.

Design of Viscous Dampers Connecting Adjacent Buildings

The second application example consists of the design of linear viscous dampers connecting two adjacent buildings with the same properties as the Buildings A and B considered in the previous application example. Two design options were investigated also in this case: (1) viscous dampers with uniform properties connecting the four lower stories of the two buildings (uniform-distribution case), which corresponds to a single design variable, c_d; and (2) viscous dampers with variable properties connecting the four lower stories of the two buildings (variable-distribution case), which corresponds to four design variables, $\mathbf{d} = [c_{d1}, c_{d2}, c_{d3}, c_{d4}]^T$. Both deterministic and uncertain structural models were investigated. In the latter case, following Sues et al. (1985), the lumped mass and story stiffness of each building were assumed to be lognormally distributed, with mean value equal to the value initially assumed

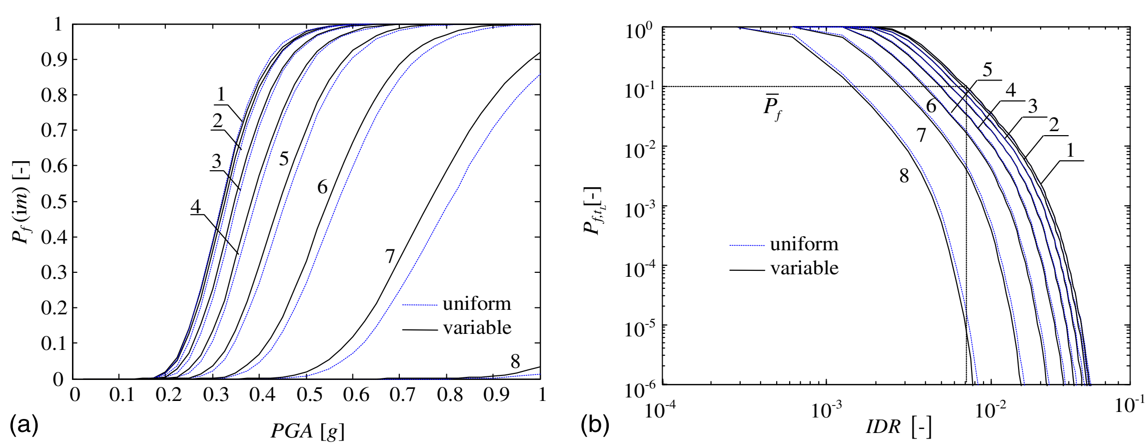

Fig. 3. Failure probabilities for Building A: (a) component fragility curves; (b) component failure probabilities during the design life

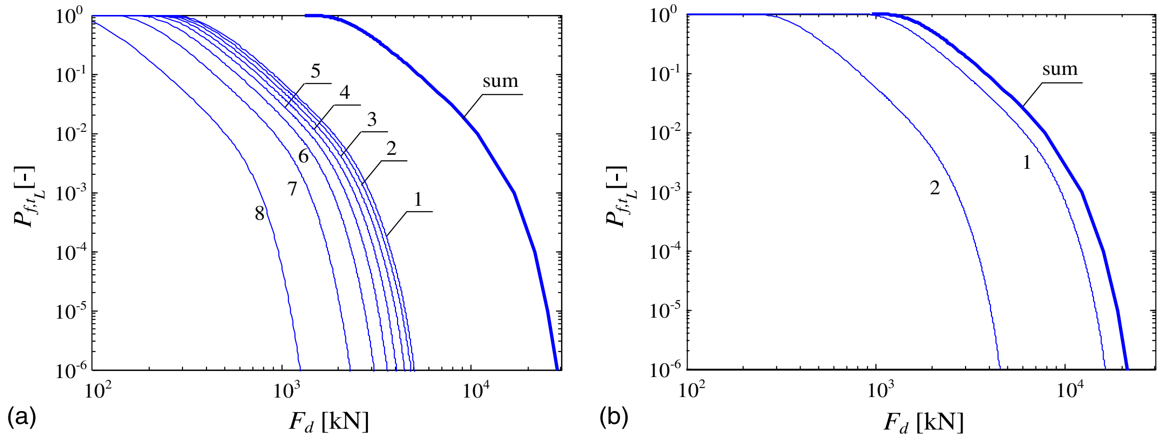

Fig. 4. Probability of dampers' forces exceeding prescribed values in 50 years in Building A: (a) uniform-distribution case; (b) variable-distribution case

as deterministic and coefficients of variation equal to 0.10 and 0.11, respectively. The damping ratios used to build the Rayleigh damping matrixes for the two separate systems were also modeled as random variables with mean value equal to 2% and with coefficient of variation equal to 0.65. Perfect correlation was assumed between the lumped masses and story stiffness within each building. Thus, six random variables were used to describe the MPU (i.e., story mass, story stiffness, and damping ratio for each building). Similar to Tubaldi et al. (2014), the assumption of perfect correlation is preferred here to a more-rigorous random field approach (Lee and Mosalam 2004) in order to avoid difficulties due to the lack of data associated with the correlation lengths of the pertinent random fields. This assumption (which corresponds to assuming that the correlation length for the corresponding random field is larger than the dimension of the structural system considered) provides an upper bound of the MPU effects on the failure probability and, thus, on the reliability-based design results.

Fifty samples of structural models were generated by means of LHS (Iman and Conover 1980), in order to describe with sufficient accuracy the variability of the uncertain parameters. An exterior sampling approach was adopted for the study of the uncertain structures, i.e., the same set of 50 LHS realizations of the vector of uncertain model parameters was employed at each iteration of the SIM algorithm to obtain the approximate and reference design solutions for the uncertain structural models. The selected sample number provides estimates of the failure probability at the design points corresponding to structures with uncertain parameters with coefficient of variations lower than 0.05. The results presented hereinafter correspond to a single set of parameters' samples. However, in order to assess the effect of using different samples, the reliability-based design procedure was repeated for several samples' sets and provided design results that had differences smaller than 0.5% in terms of $C(\mathbf{d}^*)$.

The probability of exceeding the immediate occupancy performance level in 50 years for the uncoupled deterministic system was estimated equal to 33.8% by using MCS. The same constraints, target performance objectives, and termination rules employed in the previous application example were adopted also here. The AN algorithm was based on the modified Vanmarcke's approximation and the assumption of perfect correlation between the failure modes. The failure probability estimates used in the SIM algorithm were obtained using MCS and 10,000 samples. Multiple start points were considered for the AN algorithm to find the global minimum of the objective function, and the design point obtained from the AN algorithm was used as a hot-start point for the SIM algorithm.

Tables 3 and 4 report the optimal design results obtained using the AN algorithm, the proposed correction formula, and the SIM algorithm for the deterministic and uncertain models, respectively. When compared to the design solution obtained using the SIM algorithm, the design solution obtained using the AN algorithm is a fair approximation for the case of deterministic models (with a 22.0% difference in the uniform distribution case and a 24.4% difference in the variable distribution case) and a very good approximation for the case of uncertain models (with a 0.8% difference in the uniform distribution case and a 2.2% difference in the variable distribution case). The proposed correction formula provides dampers' properties that are always an excellent approximation of those obtained through the SIM algorithm (with a 0.8% difference in the uniform distribution case and a 2.0% difference in the variable distribution case for the deterministic models, and a 0.8% difference in the uniform distribution case and a 0.3% difference in the variable distribution case for the uncertain models).

It is observed that, in general, the uncertainty in model parameters results in an increase of the total added damping, $C(\mathbf{d}^*)$, at the optimal design point, $\mathbf{d}^*$, due to an increase of the seismic risk

Table 3. Optimum Design Properties of Viscous Dampers Connecting Buildings A and B: Deterministic Models

Story number	Uniform distribution			Variable distribution		
	$\mathbf{d}^*_{AN}$ (kN · s/m)	$\mathbf{d}^*_{corr}$ (kN · s/m)	$\mathbf{d}^*_{SIM}$ (kN · s/m)	$\mathbf{d}^*_{AN}$ (kN · s/m)	$\mathbf{d}^*_{corr}$ (kN · s/m)	$\mathbf{d}^*_{SIM}$ (kN · s/m)
1	1,363.51	1,126.40	1,117.63	0	0	0
2	1,363.51	1,126.40	1,117.63	0	0	0
3	1,363.51	1,126.40	1,117.63	0	0	0
4	1,363.51	1,126.40	1,117.63	3,044.85	2,496.49	2,447.51
Sum	5,454.04	4,505.6	4,470.52	3,044.85	2,496.49	2,447.51
P_{f,t_L} (%)	8.92	9.98	10.00	8.97	9.98	10.00

Table 4. Optimum Design Properties of Viscous Dampers Connecting Buildings A and B: Uncertain Models

Story number	Uniform distribution			Variable distribution		
	$\mathbf{d}^*_{\mathrm{AN}}$ (kN · s/m)	$\mathbf{d}^*_{\mathrm{corr}}$ (kN · s/m)	$\mathbf{d}^*_{\mathrm{SIM}}$ (kN · s/m)	$\mathbf{d}^*_{\mathrm{AN}}$ (kN · s/m)	$\mathbf{d}^*_{\mathrm{corr}}$ (kN · s/m)	$\mathbf{d}^*_{\mathrm{SIM}}$ (kN · s/m)
1	1,363.51	1,363.51	1,353.14	0	0	0
2	1,363.51	1,363.51	1,353.14	0	0	0
3	1,363.51	1,363.51	1,353.14	0	0	0
4	1,363.51	1,363.51	1,353.14	3,044.85	2,987.68	2,979.21
Sum	5,454.04	5,454.04	5,412.56	3,044.85	2,987.68	2,979.21
P_{f,t_L} (%)	9.98	9.98	10.00	9.98	9.99	10.02

(Tubaldi et al. 2014). For both deterministic and uncertain models, the value of $C(\mathbf{d}^*)$ is significantly lower for the variable-distribution case than for the uniform-distribution case. Similar observations were already made in previous studies (Zhang and Xu 1999; Roh et al. 2011) and can be explained by considering that the energy dissipated by the damper(s) is roughly proportional to the maximum relative velocity between the stories of the two buildings. Since this relative velocity generally increases for increasing heights of the damper location, it is more efficient to locate the dampers at the upper stories of the buildings, where the peak relative velocity is expected to be the highest. At the optimal design points, the first-mode damping ratios of the deterministic models of Buildings A and B are 0.160 and 0.063, respectively, for the uniform-distribution case, and 0.155 and 0.068, respectively, for the variable-distribution case.

Fig. 5 plots the probability of exceedance in 50 years (evaluated using the SIM algorithm) for the IDRs of the first story of Buildings A and B for the two different optimal dampers' distributions. Fig. 5(a) shows the results for the case of deterministic model parameters, whereas Fig. 5(b) shows the results for the case of uncertain model parameters. In both cases, it is observed that the distribution of the peak interstory drift demand at the various stories is not significantly affected by the dampers' configuration. This result confirms that employing only a single damper at the optimal location permits achieving the same system performance of four uniformly-distributed dampers at a significantly lower value of the total added damping.

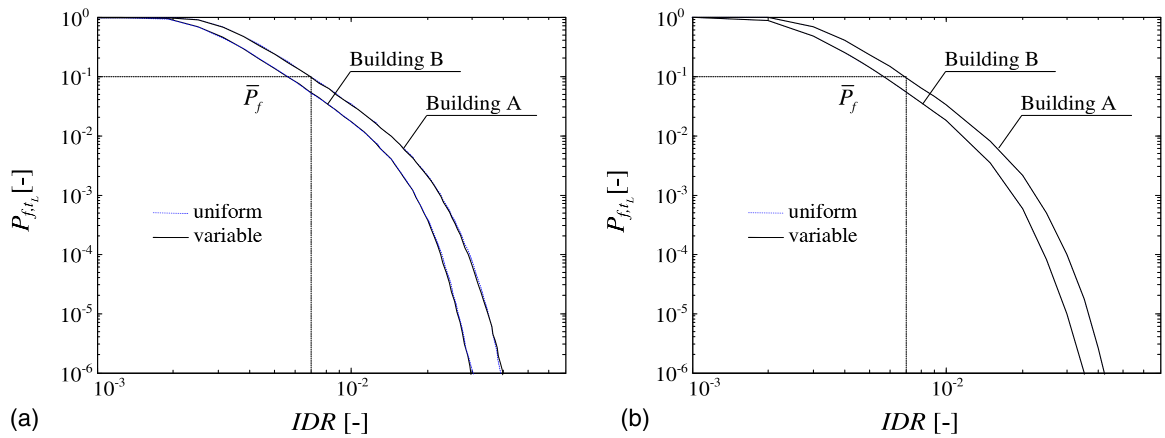

Fig. 5. Probability of exceedance in 50 years for the IDRs of the models with (a) deterministic properties; (b) uncertain properties

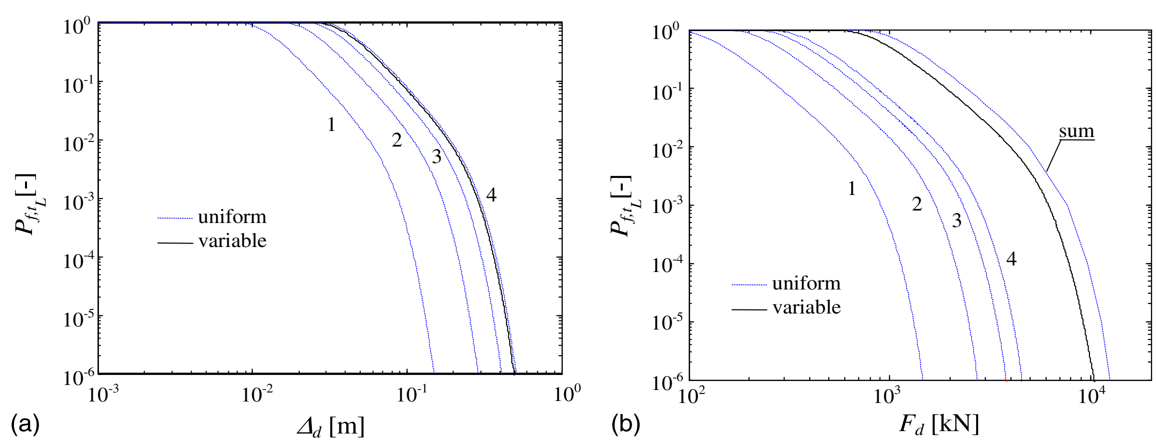

Fig. 6. Probability of exceeding in 50 years for the deterministic models of (a) dampers' strokes; (b) dampers' forces

The statistical properties of the strokes and forces produced in the dampers are also relevant design quantities that can affect the dampers' reliability and actual cost (Hwang et al. 2013). Fig. 6(a) reports the 50-year probability of exceeding a specified value of the stroke in the dampers for the optimal designs corresponding to the uniform-distribution and variable-distribution cases, relative to the deterministic model case. As expected, in the uniform-distribution case, the strokes corresponding to a given probability of exceedance increase at the upper floors. Furthermore, the single damper corresponding to the variable-distribution case (which is located at the fourth floor) shows probabilities of exceeding a given stroke that are almost coincident to those of the damper located at the fourth floor in the uniform-distribution option. Fig. 6(b) reports the 50-year probability of exceeding a specified value of the force acting in each individual damper and the sum of the forces acting in all dampers, for the optimal designs corresponding to the uniform-distribution and variable-distribution cases, relative to the deterministic model case. For a given probability of exceedance, the force acting in the single damper corresponding to the optimal design for the variable distribution case is significantly higher than the forces acting in each of the dampers for the uniform distribution case. This phenomenon is expected because the design requiring the smallest number of dampers is also characterized by the largest damper force (Hwang et al. 2013). However, for the same probability of exceedance, the sum of the forces acting in all dampers for the uniform-distribution case is significantly higher than the force acting in the single damper for the variable-distribution case. Thus, the optimal design for the variable-distribution case is more efficient than the optimal design for the uniform-distribution case also with respect to the values of the forces acting in the dampers.

Conclusions

This study presents a reliability-based methodology for the seismic design of linear viscous/viscoelastic damping devices within and/or between adjacent buildings structures. The proposed methodology, which is consistent with modern performance-based earthquake engineering frameworks, considers the uncertainty affecting both the seismic input (i.e., record-to-record variability and uncertain intensity level) and the model parameters. The optimal design of the dampers' properties and location for a target performance objective was cast in the form a constrained optimization problem with a deterministic objective function (e.g., dampers' cost) and a stochastic constraint on the system failure probability during the buildings' design life.

The general approach proposed in this study was specialized to the case of buildings with linear elastic behavior under nonstationary stochastic earthquake input, in order to take advantage of an efficient analytical technique to evaluate the system failure probability during the optimization process. The AN algorithm, previously developed by the authors to obtain the optimal separation distance to avoid seismic pounding between two adjacent buildings, was extended here to obtain an approximate solution of the optimal design of the of the dampers' properties and location. A correction formula for the solution provided by the AN algorithm was proposed to obtain an improved design solution at a small computational cost.

The proposed design methodology was illustrated by considering the retrofit of two steel buildings, modeled as shear-type multiple-degree-of-freedom linear elastic systems, by using viscous dampers. The application examples considered included individual and coupled buildings, deterministic and uncertain structural models, and viscous dampers characterized by uniform and variable properties at the various buildings' stories. The dampers' viscous constants were assumed as design variables. Their optimal values were obtained by minimizing the total added damping while satisfying the stochastic constraints on the probability of exceeding the immediate occupancy level during the buildings' design life. Based on the results obtained using the proposed design methodology for the application examples presented in this paper, the following observations were made:

1. The AN algorithm is very computationally efficient but does not strictly satisfy the stochastic constraint on the system probability of exceeding the target performance level.
2. In general, the proposed correction formula provides approximate solutions that are very close to the optimal design obtained using SIM and HYB algorithms at a very small computational cost in addition to the computational cost of the AN algorithm. This computational cost is several orders of magnitude smaller than the computational cost required by the SIM and HYB algorithms, even when the solution obtained from the AN algorithm is used as a hot-start point.
3. Model parameter uncertainty commonly produces an increase of the seismic risk estimates, which causes an increase of the total added damping for the uncertain model case when compared to the case of deterministic model.
4. For the application examples considered here, optimization of the damper location yields a significant reduction of the total added damping required to achieve a target performance level when compared to a design option in which equal dampers are used at different stories within a building or between adjacent buildings.
5. The design methodology presented in this study provides a simple yet efficient technique for the optimal design and placement of viscous/viscoelastic dissipative devices into linear elastic structural systems while controlling their seismic performance.

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